$m(t)=m_0e^{kt}$
$A(t)=A_0e^{rt}$

M133-S60: 9/15/2017

6.5 Exponential Growth and Decay

1. A population P of insects increases at a rate proportional to the current population. Suppose there are 105 insects at time $t=0$, and 575 insects seven days later.
a). Find an expression for the number $P(t)$ of insects at time $t>0$, where t is in days.

means $P(0)=105$ $\Downarrow$ $P(t)=105e^{kt}$ $k??$ $\longrightarrow$ $P(7)=105e^{7k}=575$

means $P(7)=575$

means $\frac{dP}{dt}=kP$ $\Rightarrow$ $\boxed{P(t)=P(0)e^{kt}}$

$e^{7k}=\frac{575}{105}$

$7k=\ln(575/105)$ $k=1/7\ln(575/105)$

$P(t)=105e^{1/7\ln(575/105)t}$
$=105\cdot\left(\frac{575}{105}\right)^{t/7}$

b). How many insects will there be in 51 days?

$P(51)=105\cdot\left(\frac{575}{105}\right)^{51/7}$

2. Suppose that a particular carbon isotope which is found in living trees has a half-life of 5700 years. Scientists have discovered that the charcoal from a tree burned in a volcanic eruption contains 36 percent of the isotope level that is normally found in living trees. How many years ago did the volcanic eruption occur?

$m(t)=m_0e^{kt}$

Half-life $=5700$ $\Rightarrow m(5700)=\frac{m_0}{2}$
$=m_0e^{k\cdot 5700}$ $\Big\}\Rightarrow \frac{1}{2}=e^{5700k}$; $\ln(1/2)=5700k$ $\boxed{k=-\frac{\ln(2)}{5700}}$

Charcoal discovered @ t_{now}: $\frac{36}{100}m_0$

$m(t_{now})=\frac{36}{100}m_0$
$=m_0e^{kt_{now}}$ $\Big\}\Rightarrow e^{kt_{now}}=\frac{36}{100}\Rightarrow kt_{now}=\ln(36/100)\Rightarrow t_{now}=\frac{1}{k}\ln(36/100)$

$\boxed{t_{now}=5700\,\frac{\ln(100/36)}{\ln(2)}}$ (years ago)

3. Suppose you have just placed d dollars in a bank account that pays 5 percent (annual) interest, compounded continuously.
a). How much do you have in the account after 7 years?

$A(t)=A_0e^{rt}$; $A_0=d$; $r=5/100$
$A(t)=de^{5/100t}$

$A(7)=de^{35/100}$

b). How long (in years) will it take your money to double?

$A(t)=2d$
$\cancel{d}e^{5/100t}=2\cancel{d}$
$5/100\,t=\ln 2$ $\boxed{t=20\ln(2)}$ (years)

4. Suppose that the temperature of a cup of soup obeys Newton's law of cooling. If the soup has a temperature of 175°F when served to a customer, and 5 minutes later has cooled to 145°F in a room at 64°F, how much longer must it take the soup to reach a temperature of 110°F? (how many additional minutes)

$$T(t) = T_s + (T(0) - T_s)e^{kt}$$

Soup @ 175°F when served : $T(0) = 175$
5 mins later @ 145°F : $T(5) = 145$
room @ 64°F : $T_s = 64$

$T(t) = 64 + (175-64)e^{kt}$
$= 64 + 111e^{kt}$

$T(5) = 145$
$= 64 + 111e^{5k}$ =>

$111e^{5k} = 81 \Rightarrow 5k = \ln(81/111)$

$\boxed{k = \frac{1}{5}\ln(81/111)}$

Q : Solve for t : $T(t) = 110$

$T(t) = 110$ :: $64 + 111e^{kt} = 110$
$111e^{kt} = 46$
$e^{kt} = 46/111$ $\quad kt = \ln(46/111)$
$t = 1/k \ln(46/111)$

"how many additional minutes"
=> answer is this

$$\Rightarrow \boxed{t = 5\frac{\ln(111/46)}{\ln(111/81)}} = 5\frac{\ln(46/111)}{\ln(81/111)}$$

−5

$$\text{answer} = \boxed{5\frac{\ln(111/46)}{\ln(111/81)} - 5}$$

If the same cup of 175°F soup is instead placed into a freezer set at 10°F, what is the time required for the soup to cool from 175°F to 110°F in this situation?

What changes? $T_s = 10$ => $T(t) = 10 + (175-10)e^{kt}$
$= 10 + 165e^{kt}$

Does k change? No, that constant belongs to the same soup.

$T(t) = 110$ (Solve for t)

$10 + 165e^{kt} = 110$; $e^{kt} = 100/165$; $kt = \ln(100/165)$; $t = 1/k \ln(100/165)$

$$\boxed{t = 5\frac{\ln(165/100)}{\ln(111/81)}}$$

5. Suppose that the temperature of a loaf of baked bread satisfies Newton's law of cooling. Right now the temperature of the bread is 65°C above room temperature but twenty-five minutes ago, it was 70°C above room temperature.

a). How far above room temperature will the bread be thirty-five minutes from now?

In this problem: easier to work w/ $y(t) = T(t) - T_s$ $\Rightarrow$ $\boxed{y(t) = y(0)e^{kt}}$

$\Rightarrow$ Know: $y(0) = 70 \longrightarrow y(t) = 70e^{kt}$

$y(25) = 65 \longrightarrow y(25) = 70e^{25k} = 65$; $25k = \ln(65/70)$ $\boxed{k = \frac{1}{25}\ln(65/70)}$

(a). Find $y(60) = ?$ [Why? 35 mins from now means $y(25+35)$]

$$y(60) = 70e^{60 \cdot \frac{1}{25}\ln(65/70)} = 70 \cdot \left(\frac{65}{70}\right)^{12/5}$$

b). How much greater than room temperature will the bread be 4 hours from now?

Find $y(25+240) = y(265) = ?$

$$y(265) = 70 \cdot e^{265 \cdot \frac{1}{25}\ln(65/70)} = 70 \cdot \left(\frac{65}{70}\right)^{53/5}$$

c). In how many minutes (from now) will the temperature of the bread measure 32°C above room temperature?

Solve $y(t) = 32$: $70e^{kt} = 32$; $kt = \ln(32/70)$; $t = 25\frac{\ln(32/70)}{\ln(65/70)}$

answer? not yet

"from now" : this -25

answer: $25\frac{\ln(32/70)}{\ln(65/70)} - 25$