11.6 Absolute Convergence
Ratio & Root Tests

- A series $\sum_{n=1}^{\infty} a_n$ is said to be absolutely convergent if the series $\sum_{n=1}^{\infty} |a_n|$ is convergent.

- If a series is convergent but not absolutely convergent, it is said to be conditionally convergent.

Example: The Alternating Harmonic Series is conditionally convergent:

$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{n}$ converges

$\sum_{n=1}^{\infty} \frac{1}{n}$ does not converge

Example: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ is absolutely convergent: $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series.

- If a series is absolutely convergent, then it is convergent.

Ratio Test: $\boxed{\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L}$

- If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent
- If $L > 1$ (including if $L = \infty$), then $\sum_{n=1}^{\infty} a_n$ is divergent.
- If $L = 1$: no conclusion

Root Test: $\boxed{\lim_{n\to\infty} \sqrt[n]{|a_n|} = L}$

- If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent
- If $L > 1$ (including if $L = \infty$), then $\sum_{n=1}^{\infty} a_n$ is divergent
- If $L = 1$: no conclusion