MATH 132 - Michigan State University
September 24th, 2018.

2.7 Worksheet: Rates of Change

1. A particle moves according to the law of motion

$$s(t) = t^3 - 8t^2;\ t \geq 0,$$

where t is measured in seconds and s in feet.

$S(t)$ = position
$v(t) = S'(t)$ velocity
$a(t) = v'(t) = S''(t)$ acceleration

a). Find the average velocity over the interval $[1, 2]$.

$$\frac{S(2)-S(1)}{2-1} = \frac{(8-32)-(1-8)}{1} = \boxed{-17}\ \text{ft/sec}$$

b). Find the velocity at time t.

$$v(t) = S'(t) = 3t^2 - 16t$$

c). What is the velocity after 3 seconds?

$$v(3) = 3\cdot 9 - 16\cdot 3 = \boxed{-21}\ \text{ft/sec}$$

d). What is the acceleration after 5 seconds?

$$a(t) = v'(t) = 6t - 16 \Rightarrow a(5) = 6\cdot 5 - 16 = \boxed{14}\ \text{ft/sec}^2$$

e). For $t \geq 0$, when is the particle moving in the positive direction?

→ means $v(t) > 0$

$$v(t) = 3t^2 - 16t = t(3t-16)$$

Roots: $0,\ 16/3$

	0		16/3	
t	− − 0	+	+	+ +
$3t-16$	− −	−	0	+ + +
$v(t)$	+ 0	−	0	+ +

$$\boxed{\left(\tfrac{16}{3}, \infty\right)}$$

2. A particle moves according to the law of motion

$$s = t^3 - 8t^2 + 4t,\ t \geq 0.$$

For $t \geq 0$, when is the particle moving in the positive direction?

$$v(t) = 3t^2 - 16t + 4$$

Roots:

$$\frac{16 \pm \sqrt{16^2 - 16\cdot 3}}{6} = \frac{16 \pm 4\sqrt{16-3}}{6} = \boxed{\frac{8 \pm 2\sqrt{13}}{3}}$$

	0		$\frac{8-2\sqrt{13}}{3}$		$\frac{8+2\sqrt{13}}{3}$	
$v(t)$	+	+ +	0	− − − −	0	+ + +

$$\boxed{\left(0, \frac{8-2\sqrt{13}}{3}\right) \cup \left(\frac{8+2\sqrt{13}}{3}, \infty\right)}$$

3. A particle moves according to the law of motion

$$s(t) = \frac{4}{t^2} - \frac{2}{t},\ t \geq 0.$$

For $t \geq 0$, when is the particle moving in the negative direction?

→ means $v(t) < 0$

$$v(t) = \frac{-8}{t^3} + \frac{2}{t^2} = \frac{2t-8}{t^3}$$

$2t-8$ → negative for $t \in (0,4)$

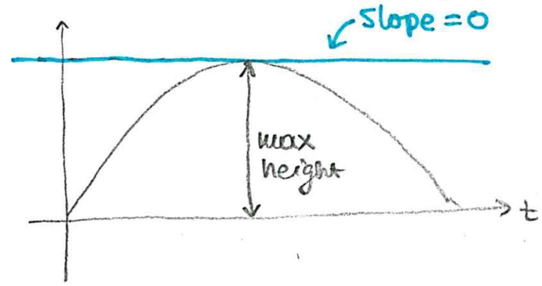
t^3 → always positive for $t > 0$

$$\boxed{(0,4)}$$

t	0		4	
$2t-8$		− −	0	+ + +

4. If an arrow is shot straight upward on some moon with a velocity of 40m/s, its height in meters after t seconds is given by

$$s(t) = 40t - 10t^2.$$



a). At what time will the arrow reach its maximum height?

Max height is achieved when $v(t)=0$.

$v(t) = 40 - 20t \Rightarrow v(t)=0$ when $\boxed{t=2}$

b). How long will it take for the arrow to return and hit the moon?

The arrow is on the ground when $s(t)=0$.

$s(t) = 40t - 10t^2 = 10t(4-t) \Rightarrow s(t)=0$ at $t=0$ (launch) & $\boxed{t=4}$ (hits ground)

c). With what velocity will the arrow hit the moon?

$v(4) = 40 - 80 = \boxed{-40}$ m/s

5. At time t seconds, the position of a body moving along the s-axis is

$$s(t) = t^3 - 6t^2 + 9t \text{ (meters)} = f(t)$$

a). Find the acceleration of the body each time the velocity is zero.

$v(t) = 3t^2 - 12t + 9 = 3(t^2-4t+3) = 3(t-1)(t-3)$

$a(t) = 6t - 12$

$v(t)=0$ for $t=1, t=3$

$a(1) = 6-12 = -6$

$a(3) = 18-12 = 6$

$\boxed{-6, 6}$

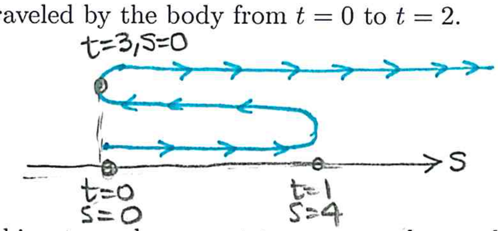
b). Find the speed of the body each time the acceleration is zero.

→ Absolute value of velocity!

$a(t)=0$ when $t=2$; $v(2) = 12-24+9 = -3 \Rightarrow$ speed$(2) = \boxed{3}$

c). Find the total distance traveled by the body from $t = 0$ to $t = 2$.

t	0		1		3	
v(t)	+	+	0	−	0	+ + +
s(t)	↗		rest ↘		rest ↗	



From [0,1]: $|f(1)-f(0)| = 4$ (f(1)=4, f(0)=0)

From [1,2]: $|f(2)-f(1)| = |-2| = 2$ (f(2)=2, f(1)=4)

Total distance: 6

6. Explorers on a planet with a thin atmosphere want to measure the acceleration of gravity at its surface. So they use a spring gun to launch a ball bearing vertically upward from the surface with a launch velocity of $16m/s$. If we denote by g_s the acceleration of gravity in that planet, in m/s^2, then the ball bearing will move following the formula

$$s(t) = 16t - \frac{g_s}{2}t^2.$$

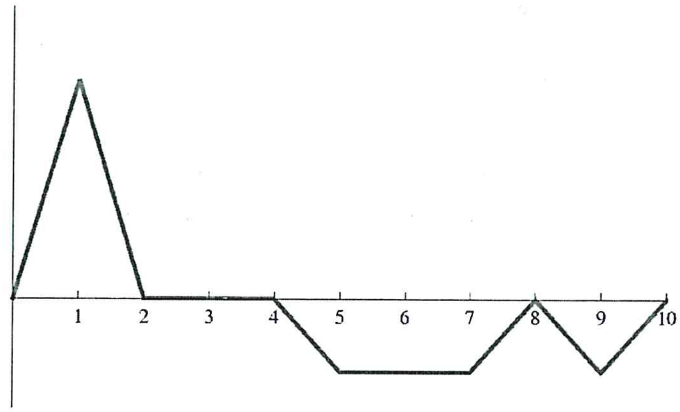
If the ball bearing reaches its maximum height 33 seconds after being launched, what is the value of g_s?

→ means $v(33)=0$

$v(t) = s'(t) = 16 - \frac{g_s}{2}\cdot 2t = 16 - g_s \cdot t$

$v(33)=0 \Rightarrow 16 - g_s\cdot 33 = 0 \Rightarrow g_s = \frac{16}{33}$ m/s²

7. The figure shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in the closed interval $[0, 10]$.



a). When does the particle move forward?

$(0,2)$

A particle moves forward (in the positive direction) when $v(t) > 0$.

b). When does the particle move backward? → $v(t) < 0$

$(4,8) \cup (8,10)$

c). When does the particle speed up?

$(0,1) \cup (4,5) \cup (8,9)$

A particle speeds up when: velocity is ⊕ and ↗; velocity is ⊖ and ↘ (when velocity & acceleration have the same sign).

d). When does the particle slow down?

$(1,2) \cup (7,8) \cup (9,10)$

A particle slows down when: velocity is ⊕ and ↘; velocity is ⊖ and ↗ (when velocity & acceleration have opposite signs)

e). When is the particle?s acceleration positive? → when v is ↗

$(0,1) \cup (7,8) \cup (9,10)$

f). When is the particle?s acceleration negative? → when v is ↘

$(1,2) \cup (4,5) \cup (8,9)$

g). When is the particle?s acceleration zero? → when v is constant

$(2,4) \cup (5,7)$

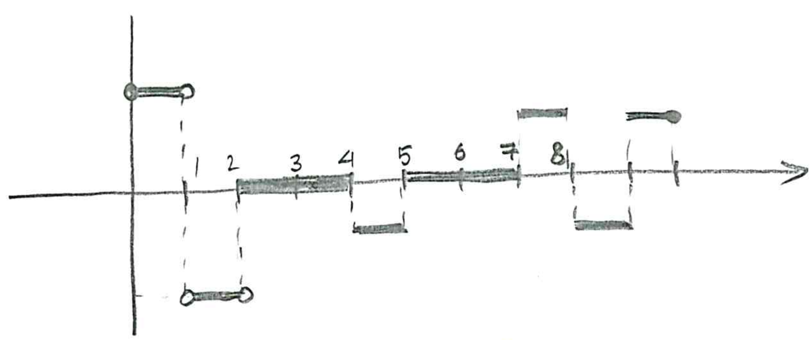
h). When does the particle move at its greatest speed?

$t = 1$

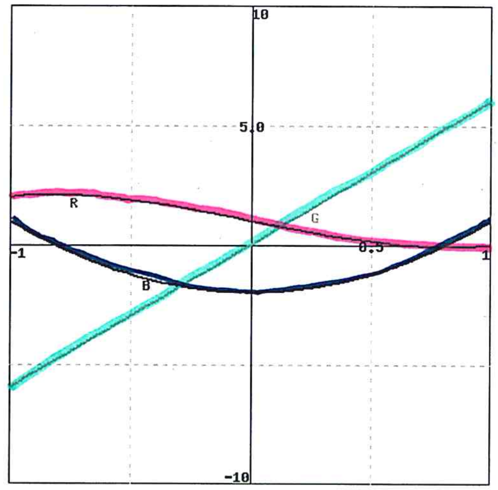
i). When does the particle stand still for more than an instant?

$[2,4]$.

Acceleration:



8. In the picture below identify the graphs B (blue), R (red) and G (green) as the graphs of a position function, the corresponding velocity function, and the corresponding acceleration function.



green = (blue)'

=> either (pos - B, vel - G, acc - R) X

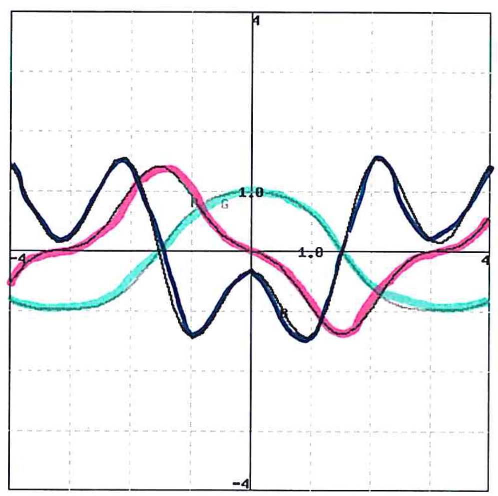
or (pos - R, vel - B, acc - G) ✓

The position function is the graph in RED

The velocity function is the graph in BLUE

The acceleration function is the graph in GREEN

9. In the picture below identify the graphs B (blue), R (red) and G (green) as the graphs of a position function, the corresponding velocity function, and the corresponding acceleration function.



Either (red = (green)')

or red = (blue)' ← NO

=> either (pos - G, vel - R, acc - B) ✓

or (pos - B, vel - G, acc - R) X

The position function is the graph in Green

The velocity function is the graph in Red

The acceleration function is the graph in Blue