Implicit Differentiation - Extra Problems

1. $\dfrac{1}{x} + \dfrac{1}{y} = 4$; $y(5) = \dfrac{5}{19}$. Find $y'(5)$.

2. $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$. Find the equation of the tangent line at the point $(-8,\ 3\sqrt{3})$.

3. $x^2y + y^6 - 5x = 16$; $\dfrac{dy}{dx} = ?$

4. $x^3y + 3xy^3 = x + y$; $\dfrac{dy}{dx} = ?$

5. $y^{-7/2} + x^{7/2} = 1$; $\dfrac{dy}{dx} = ?$

Implicit Differentiation – Practice Problems

① $\frac{1}{x}+\frac{1}{y}=4;\quad y(5)=\frac{5}{19}\quad ; y'(5)?$

$$-\frac{1}{x^2}-\frac{1}{y^2}y'=0\ ;\quad \boxed{y'=\frac{-y^2}{x^2}}\ \Rightarrow y'(5)=\frac{-y^2(5)}{5^2}=\frac{-\frac{25}{192}}{25}=\frac{-1}{192}$$

② Tangent line to curve $\frac{x^2}{16}-\frac{y^2}{9}=1$ at $(-8, 3\sqrt{3})$

$$\frac{d}{dx}\left(\frac{x^2}{16}-\frac{y^2}{9}\right)=\frac{d}{dx}(1)$$

$$\frac{2x}{16}-\frac{2y}{9}\cdot\frac{dy}{dx}=0\ ;\ \frac{2y}{9}\frac{dy}{dx}=\frac{x}{8}\ ;\ \boxed{\frac{dy}{dx}=\frac{9x}{16y}}$$

$$\left.\frac{dy}{dx}\right|_{x=-8}=\frac{-9\cdot 8}{16\cdot 3\sqrt{3}}=\frac{-24}{16\sqrt{3}}=\frac{-3}{2\sqrt{3}}=\frac{-\sqrt{3}}{2}$$

$$y-3\sqrt{3}=\frac{-\sqrt{3}}{2}(x+8);\quad \boxed{y=\frac{-\sqrt{3}}{2}(x+8)+3\sqrt{3}}\ \text{or}\ \boxed{y=\frac{-\sqrt{3}}{2}x-\sqrt{3}}$$

③ $x^2y+y^6-5x=16;\quad \frac{dy}{dx}=?$

$$2x\cdot y+x^2\cdot\frac{dy}{dx}+6y^5\frac{dy}{dx}-5=0$$

$$(x^2+6y^5)\frac{dy}{dx}=5-2xy\ \Rightarrow\ \frac{dy}{dx}=\frac{5-2xy}{x^2+6y^5}$$

④ $x^3y+3xy^3=x+y\ ;\ \frac{dy}{dx}=?$

$$3x^2y+x^3\frac{dy}{dx}+3y^3+3x\cdot 3y^2\frac{dy}{dx}=1+\frac{dy}{dx}$$

$$(x^3+9xy^2)\frac{dy}{dx}=1+\frac{dy}{dx}-(3x^2y+3y^3)$$

$$(x^3+9xy^2-1)\frac{dy}{dx}=1-3x^2y-3y^3\qquad \boxed{\frac{dy}{dx}=\frac{1-3x^2y-3y^3}{x^3+9xy^2-1}}$$

⑤ $y^{-7/2}+x^{7/2}=1$

$$-\frac{7}{2}y^{-9/2}\frac{dy}{dx}+\frac{7}{2}x^{5/2}=0\ \Rightarrow\ \frac{dy}{dx}=\frac{\cancel{\frac{7}{2}}x^{5/2}}{\cancel{\frac{7}{2}}y^{-9/2}}\Rightarrow\ \boxed{\frac{dy}{dx}=x^{5/2}y^{9/2}}$$