

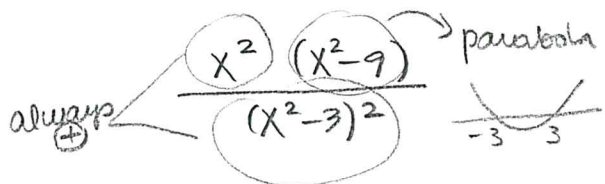
Quiz 7 - Solutions.

$$f(x) = \frac{x^3}{x^2-3}$$

① Domain: $(-\infty, -\sqrt{3}) \cup (-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, \infty)$.

② Monotonicity: $f'(x) = \frac{3x^2(x^2-3) - x^3(2x)}{(x^2-3)^2} = \frac{3x^4 - 9x^2 - 2x^4}{(x^2-3)^2}$

$$= \frac{x^4 - 9x^2}{(x^2-3)^2} = \frac{x^2(x^2-9)}{(x^2-3)^2}$$



x	-3	$-\sqrt{3}$	$\sqrt{3}$	3		
$f'(x)$	$+$	0	$-$	0	$+$	$+$
	\nearrow	\rightarrow	\searrow	\searrow	\nearrow	\nearrow

$f \uparrow$ on $(-\infty, -3) \cup (3, \infty)$

$f \downarrow$ on $(-3, -\sqrt{3}) \cup (-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, 3)$.

③ Concavity: $f''(x) = \frac{(4x^3 - 18x)(x^2-3)^2 - (x^4 - 9x^2) \cdot 2(x^2-3) \cdot 2x}{(x^2-3)^4}$

$$= \frac{(4x^3 - 18x)(x^2-3) - 4x(x^4 - 9x^2)}{(x^2-3)^3}$$

$$= \frac{6x(x^2+9)}{(x^2-3)^3}$$

$$4x^5 - 12x^3 - 18x^3 + 54x$$

$$- 4x^5 + 36x^3$$

$$= 6x^3 + 54x$$

$$= 6x(x^2+9)$$

x	$-\sqrt{3}$	0	$\sqrt{3}$
$6x(x^2+9)$	$-$	0	$+$
$(x^2-3)^3$	$+$	0	$+$
f''	$-$	$+$	$-$
	\cap	\cup	\cap

f is \cap on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

f is \cup on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

④ Vertical Asymptotes: $X = \sqrt{3}$, $X = -\sqrt{3}$

$$f(x) = \frac{x^3}{x^2 - 3}$$



$$\lim_{x \rightarrow -\sqrt{3}^-} f(x) = \frac{\ominus}{0^+} = -\infty$$

$$\lim_{x \rightarrow -\sqrt{3}^+} f(x) = \frac{\ominus}{0^-} = +\infty$$

$$\lim_{x \rightarrow \sqrt{3}^-} f(x) = \frac{\oplus}{0^-} = -\infty$$

$$\lim_{x \rightarrow \sqrt{3}^+} f(x) = \frac{\oplus}{0^+} = +\infty$$

Horizontal Asymptotes / Limits at ∞ :

$$\lim_{x \rightarrow \infty} \frac{x^3}{x^2 - 3} = \lim_{x \rightarrow \infty} \frac{x}{1 - \frac{3}{x^2}} = \oplus\infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^3}{x^2 - 3} = \lim_{x \rightarrow -\infty} \frac{-x}{1 - \frac{3}{x^2}} = \ominus\infty$$

(no horizontal asymptotes).

Slant Asymptotes:

$$x^3 = x(x^2 - 3) + 3x \Rightarrow f(x) = \frac{x(x^2 - 3) + 3x}{x^2 - 3} = x + \frac{3x}{x^2 - 3}$$

$$\boxed{y = x}$$

$$\frac{3x}{x^2 - 3} \begin{matrix} \xrightarrow{0} \\ x \rightarrow \infty \\ x \rightarrow -\infty \end{matrix}$$

x	$-\infty$	-3	$-\sqrt{3}$	0	$\sqrt{3}$	3	∞
$f'(x)$	+	+	+	0	-	-	-
$f''(x)$	-	-	-	-	+	0	-
$f(x)$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$

