Quiz 5 Solutions

① (2pts). $\lim_{x\to7} \frac{3x-9}{x^2-10x+21} = \lim_{x\to7} \frac{3\cancel{(x-3)}}{\cancel{(x-3)}(x-7)} = \lim_{x\to7} \frac{3}{x-7}$ $\underline{\underline{\text{DNE}}}$ $\left(\frac{3}{0}\right)$

$\lim_{x\to7^-} \frac{3}{x-7} = -\infty$; $\lim_{x\to7^+} \frac{3}{x-7} = +\infty$.

1pt. : Side limits
1pt. : final result

② (2pts) $\lim_{x\to1} \frac{x^2+7x-8}{x^2-x} = \lim_{x\to1} \frac{\cancel{(x-1)}(x+8)}{x\cancel{(x-1)}} = 9$

1pt.: Simplification /factorization
1pt.: final answer

③ (2pts) $\lim_{x\to1} \frac{x^2-1}{|x-1|}$

$$|x-1| = \begin{cases} -(x-1), & \text{if } x-1<0 \text{ or } x<1 \\ +(x-1), & \text{if } x-1\geq0 \text{ or } x\geq1 \end{cases}$$

$\lim_{x\to1^-} \frac{\cancel{(x-1)}(x+1)}{-\cancel{(x-1)}} = -2$ $\quad \lim_{x\to1^+} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = 2$.

1pt.: Side limits
1pt.: final result

④ (4pts) $f(x) = \sqrt{x-1}$

$$\lim_{h\to0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h} \quad \Big| \cdot \frac{\sqrt{x+h-1} + \sqrt{x-1}}{\sqrt{x+h-1} + \sqrt{x-1}}$$

2pts : correct defn. limit
1pt.: mult. by conjugate
1pt. : final answer.

$$= \lim_{h\to0} \frac{x+h-1-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$$

$$= \lim_{h\to0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h-1}+\sqrt{x-1}\right)} = \lim_{h\to0} \frac{1}{\sqrt{x+h-1}+\sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$$