NAME: Solutions

MATH 132 - Michigan State University
September 14th, 2018.

Quiz 2

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. [4 points] Consider the function:

$$f(x) = \begin{cases} x^2 - 5x + 4, & \text{if } x < 0 \\ 10, & \text{if } x = 0 \\ \frac{4x}{x^2+x}, & \text{if } x > 0. \end{cases}$$

a). [3 pts.] Find the limits:

$$\lim_{x\to 0_-} f(x) = \lim_{x\to 0_-} (x^2-5x+4) = \boxed{4}$$ 1pt.

$$\lim_{x\to 0_+} f(x) = \lim_{x\to 0_+} \frac{4}{x+1} = \boxed{4}$$ 2pts.

b). [1 pt.] Is the function f continuous at 0? If not, state the type of discontinuity.

$\lim_{x\to 0} f(x) = 4$ but $f(0) = 10 \Rightarrow f$ is not continuous at 0 ; removable discontinuity.

2. [6 points] Consider the function

$$f(x) = 3\sqrt{x} - 5.$$

a). [3 pts.] Simplify as much as possible the quotient

$$\frac{f(4+h)-f(4)}{h} = \frac{(3\sqrt{4+h}-\cancel{5}) - (3\sqrt{4}-\cancel{5})}{h}$$ 1pt. (plugging in)

$$= \frac{3(\sqrt{4+h}-\sqrt{4})}{h} \cdot \frac{\sqrt{4+h}+\sqrt{4}}{\sqrt{4+h}+\sqrt{4}}$$ 1pt. (mult. by conjugate)

$$= \frac{3((\cancel{4}+h)-\cancel{4})}{h(\sqrt{4+h}+\sqrt{4})} = \boxed{\frac{3}{\sqrt{4+h}+\sqrt{4}}}$$ 1pt. (final reduction)

b). [2 pts.] Use the results in part a). to find $f'(4)$.

$$\lim_{h\to 0} \frac{f(4+h)-f(4)}{h} = \lim_{h\to 0} \frac{3}{\sqrt{4+h}+2} = \frac{3}{2+2} = \boxed{\frac{3}{4}}$$

1pt.-correct limit def.
1pt.-numerical res.

c). [1 pt.] Write the equation of the line tangent to the curve $y = f(x)$ at the point $(4, f(4))$.

$f(4) = 1$ Point: $(4,1)$ Slope: $3/4$

$\boxed{y - 1 = \frac{3}{4}(x-4)}$ or $\boxed{y = \frac{3}{4}x - 2}$ 1pt.

Bonus: [1 pt.] Find

$$\lim_{x\to\frac{\pi}{4}} \cos\left(5x - \underbrace{\sin(4x)}\right) = \cos\left(5\cdot\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$$

$\downarrow$

$\sin(4\cdot\frac{\pi}{4}) = \sin(\pi) = 0$