

Quiz 10

1. Find the absolute maxima and minima for the function $f(x) = x + \frac{1}{x}$ on the interval $[\frac{1}{2}, 2]$.

(1pt.) $f'(x) = 1 - \frac{1}{x^2}$

(1pt.) $\left\{ \begin{array}{l} f'(x) = 0 \Rightarrow 1 - \frac{1}{x^2} = 0 \Rightarrow 1 = x^2 \Rightarrow x = \pm 1 \\ \text{Only } x=1 \text{ is a critical number in } [\frac{1}{2}, 2]. \end{array} \right.$

(3pts.) $\left\{ \begin{array}{l} f(\frac{1}{2}) = \frac{1}{2} + 2 = \left(\frac{5}{2}\right) \leftarrow \text{max} \\ f(1) = 1 + 1 = \left(2\right) \leftarrow \text{min} \\ f(2) = 2 + \frac{1}{2} = \left(\frac{5}{2}\right) \end{array} \right.$

Absolute min: 2

Absolute max: $\frac{5}{2}$

2. Find the limit:

$$\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 5x} - 2x)$$

(1pt.) $= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 5x} - 2x) \cdot \frac{\sqrt{4x^2 + 5x} + 2x}{\sqrt{4x^2 + 5x} + 2x}$

(1pt.) $= \lim_{x \rightarrow \infty} \frac{4x^2 + 5x - 4x^2}{\sqrt{4x^2 + 5x} + 2x}$

(1pt.) $= \lim_{x \rightarrow \infty} \frac{5x}{\sqrt{4x^2 + 5x} + 2x} \quad \left| \begin{array}{l} \frac{1}{x} \\ \frac{1}{x} \end{array} \right.$

(1pt.) $= \lim_{x \rightarrow \infty} \frac{5}{\sqrt{4 + \frac{5}{x}} + 2} = \frac{5}{\sqrt{4+2}} = \left(\frac{5}{4}\right)$ (1pt.)