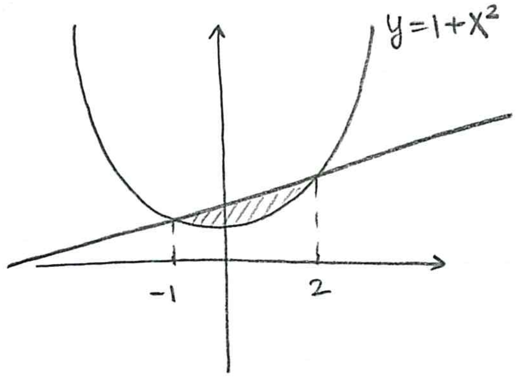
① Area b/w curves $y = 1 + x^2$, $y = 3 + x$




Intersection: $1 + x^2 = 3 + x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\underline{x = 2}, \ \underline{x = -1}$

$$A = \int_{-1}^{2} \left( (3+x) - (1+x^2) \right) dx$$

$$= \int_{-1}^{2} \left( -x^2 + x + 2 \right) dx = \left( -\frac{x^3}{3} + \frac{x^2}{2} + 2x \right) \Bigg|_{-1}^{2}$$

$$= \left( -\frac{8}{3} + \frac{4}{2} + 4 \right) - \left( \frac{1}{3} + \frac{1}{2} - 2 \right)$$

$$= -\frac{9}{3} + \frac{3}{2} + 6 = \frac{3}{2} + 3 = \frac{9}{2}$$

② Area b/w curves $y = 9 - \frac{x^2}{4}$ and $y = 6 - x$.

Intersection: $9 - \frac{x^2}{4} = 6 - x$

$\frac{x^2}{4} - x - 3 = 0 \quad | \times 4$

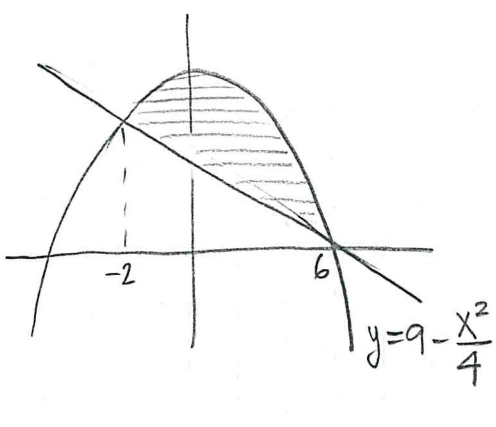
$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0 \Rightarrow x = -2, 6$

$x = -2 \Rightarrow y = \begin{cases} 9 - \frac{4}{4} = 8 \checkmark \\ 6 + 2 = 8 \checkmark \end{cases}$

$x = 6 \Rightarrow y = \begin{cases} 9 - \frac{36}{4} = 0 \checkmark \\ 6 - 6 = 0 \checkmark \end{cases}$

Points: $(-2, 8)$ & $(6, 0)$

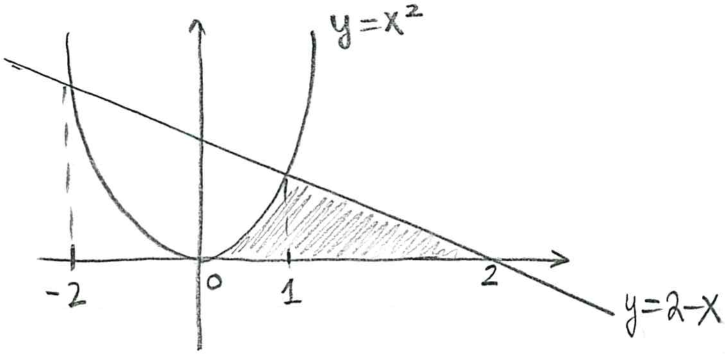



$$A = \int_{-2}^{6} \left( \left(9 - \frac{x^2}{4}\right) - (6 - x) \right) dx$$

$$= \int_{-2}^{6} \left( 3 + x - \frac{x^2}{4} \right) dx$$

$$= \boxed{\left( 3x + \frac{x^2}{2} - \frac{x^3}{12} \right) \Bigg|_{-2}^{6}} = \frac{64}{3}$$

(3) Area bounded by curves $y=x^2$, $y=2-x$ and $y=0$, in Quad I.

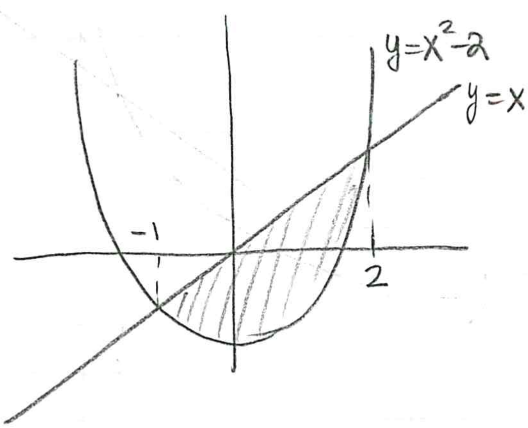



$$x^2=2-x$$
$$x^2+x-2=0$$
$$(x+2)(x-1)=0$$

$$A=\int_0^1 x^2dx+\int_1^2(2-x)\,dx = \frac{x^3}{3}\Big|_0^1+\left(2x-\frac{x^2}{2}\right)\Big|_1^2$$
$$=\frac{1}{3}+(4-2)-\left(2-\frac{1}{2}\right)=\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$$

(4) Area bounded by $y=x^2-2$ and $y=x$.

$$x^2-2=x \Rightarrow x^2-x-2=0$$
$$(x-2)(x+1)=0 \Rightarrow x=-1,2$$




$$A=\int_{-1}^{2}\left(x-(x^2-2)\right)dx$$
$$=\left(\frac{x^2}{2}-\frac{x^3}{3}+2x\right)\Big|_{-1}^{2}$$
$$=\left(\frac{4}{2}-\frac{8}{3}+4\right)-\left(\frac{1}{2}+\frac{1}{3}-2\right)$$
$$=6-\frac{8}{3}-\frac{1}{2}-\frac{1}{3}+2=8-3-\frac{1}{2}=5-\frac{1}{2}=\frac{9}{2}$$

5 a) $y = \cos(\sqrt{x})$

$$\frac{dy}{dx} = -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

f) $y = \tan(3x)\sec\left(\frac{1}{x}\right)$

$$\frac{dy}{dx} = \sec^2(3x)\cdot 3\cdot \sec\left(\frac{1}{x}\right) + \\ + \tan(3x)\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\cdot \frac{-1}{x}$$

b) $x^3 + y^3 = 4$

$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} = -3x^2 \Rightarrow \frac{dy}{dx} = \frac{-x^2}{y^2}$$

c) $y = x^{-1/6} \sin(7\pi x)$

$$\frac{dy}{dx} = \frac{-1}{6} x^{-7/6} \sin(7\pi x) + x^{-1/6} \cos(7\pi x)\cdot 7\pi$$

d) $y = \sin(3x + 4y)$

$$\frac{dy}{dx} = \cos(3x+4y)\cdot\left(3 + 4\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = 3\cos(3x+4y) + 4\cos(3x+4y)\cdot\frac{dy}{dx}$$

$$\left(1 - 4\cos(3x+4y)\right)\frac{dy}{dx} = 3\cos(3x+4y)$$

$$\frac{dy}{dx} = \frac{3\cos(3x+4y)}{1-4\cos(3x+4y)}$$

e) $x = \sqrt{x^2+y^2}$

$$1 = \frac{1}{2\sqrt{x^2+y^2}}\left(2x + 2y\frac{dy}{dx}\right) = \frac{x}{\sqrt{x^2+y^2}} + \frac{y}{\sqrt{x^2+y^2}}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 - \frac{x}{\sqrt{x^2+y^2}}}{\frac{y}{\sqrt{x^2+y^2}}} = \frac{\sqrt{x^2+y^2} - x}{y}$$

g) $\sqrt{x} = \cos y + \sin x$

$$\frac{1}{2\sqrt{x}} = -\sin y\cdot\frac{dy}{dx} + \cos x \Rightarrow \sin y\cdot\frac{dy}{dx} = \cos x - \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x - \frac{1}{2\sqrt{x}}}{\sin y}.$$

⑥ $\int x\cos(3x^2+5)\,dx$

$u = 3x^2+5$
$du = 6x\,dx$
$\frac{1}{6}du = x\,dx$

$$= \int \cos(u)\,\frac{1}{6}du = \frac{1}{6}\sin(u) + C$$
$$= \boxed{\frac{1}{6}\sin(3x^2+5)+C}$$

⑦ $\int 2x\sqrt{1+x^2}\,dx$

$u = 1+x^2$
$du = 2x\,dx$

$$= \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(1+x^2)+C$$

⑧ $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}dx$
$2du = \frac{1}{\sqrt{x}}dx$

$$\int \sin(u)\,2du = -2\cos(u)+C$$
$$= \boxed{-2\cos(\sqrt{x})+C}$$

⑨ $\int \sin(2x)\cos(2x)\,dx$

$u = \sin(2x)$
$du = 2\cos(2x)dx$
$\frac{1}{2}du = \cos(2x)dx$

OR

$u = \cos(2x)$
$du = -2\sin(2x)\,dx$
$-\frac{1}{2}du = \sin(2x)dx$

$$\Rightarrow \int u\cdot\frac{1}{2}du = \frac{u^2}{4}+C$$
$$= \frac{\sin^2(2x)}{4}+C$$

$\longleftarrow$ equivalent $\longrightarrow$

$$\Rightarrow \int u\cdot\frac{-1}{2}du = \frac{-u^2}{4}+C$$
$$= -\frac{\cos^2(2x)}{4}+C$$

⑩ $\int \sin(3x)\cos^9(3x)\,dx$

$u = \cos(3x)$
$du = -3\sin(3x)\,dx$
$-\frac{1}{3}du = \sin(3x)\,dx$

$$\int u^9\cdot\frac{-1}{3}du = \frac{-1}{3}\,\frac{u^{10}}{10}+C$$
$$= -\frac{\cos^{10}(3x)}{30}+C$$