(1) $\lim_{t\to -3} \frac{6+4t}{t^2+1} = \frac{6-12}{9+1} = \frac{-6}{10} = \frac{-3}{5}$

(2) $\lim_{x\to 8} \frac{x^2-7x-8}{8-x} \overset{0/0}{=} \lim_{x\to 8} \frac{\cancel{(x-8)}(x+1)}{-\cancel{(x-8)}} = \lim_{x\to 8}\left(-(x+1)\right) = -9$

(3) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4} \frac{\cancel{\sqrt{x}-2}}{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{2+2} = \frac{1}{4}$

(4) $\lim_{x\to 0} \frac{\sin(3x)}{x} = \lim_{x\to 0} \underbrace{\frac{\sin(3x)}{3x}}_{\to 1} \cdot 3 = 3$

$\lim_{a\to 0} \frac{\sin(a)}{a} = 1$

(5) $\lim_{x\to 0} \frac{\sin(2x)}{4x} = \lim_{x\to 0} \underbrace{\frac{\sin(2x)}{2x}}_{\to 1} \cdot \frac{1}{2} = \frac{1}{2}$

(6) $\lim_{x\to 1} \frac{x^{63}-1}{x-1} = g'(1)$, where $g(x)=x^{63}$

$= 63$ $\qquad g'(x) = 63x^{62}$

$g'(a) = \lim_{x\to a} \frac{g(x)-g(a)}{x-a}$

(7) $\lim_{x\to -1} \frac{x^{28}-1}{x+1} = g'(-1)$, where $g(x)=x^{28}$

$= -28$ $\qquad g'(x) = 28x^{27}$

(8) $\lim_{h\to 0} \frac{\sin(\pi+h)}{h} = g'(\pi)$, where $g(x)=\sin(x)$

$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$= \lim_{h\to 0} \frac{\sin(\pi+h) - \overbrace{\sin(\pi)}^{0}}{h} = \sin'(\pi) = \cos(\pi) = -1$

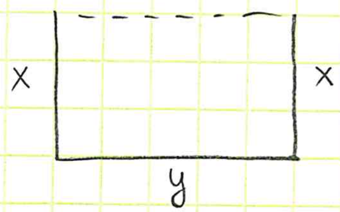
(9) $\lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4}+h\right)-1}{h} = g'(\pi/4)$ where $g(x) = \tan(x)$

$= \sec^2(\pi/4) = \frac{1}{1/2} = 2$ $\qquad g'(x) = \sec^2(x)$

(10) $\lim_{x\to 2} \frac{x^2+5x-14}{x^2-2x} = \lim_{x\to 2} \frac{(x+7)\cancel{(x-2)}}{x\cancel{(x-2)}} = \frac{9}{2}$

Optimization :

(11) Rectangular field; fence; 500 ft; one side no fencing; largest area?



$y = 500 - 2x$

Perimeter $= 2x + y = 500$ (Constraint)

Area $= xy$ (Optimize -> maximize)

$$A(x) = xy$$
$$= x(500-2x)$$
$$= 500x - 2x^2$$

$A'(x) = 500 - 4x$

$+$ $-$

$A'(x) = 0 \Rightarrow x = \frac{500}{4} = 125$

$\Rightarrow y = 500 - 2\cdot 125 = 250$

width $= y = 250$
height $= x = 125$

Justify :

x	125
$A'(x)$	$+ \; + \; 0 \; - \; - \; -$
$A(x)$	$\nearrow$ max $\searrow$

(12) Find two positive integers such that their sum is 10, and the sum of their squares is (a) minimal
(b) maximal.

Two positive integers x, y s.t. $x + y = 10 \Rightarrow y = 10 - x \Rightarrow$ range for x: $0 \le x \le 10$

$$S(x) = x^2 + y^2 = x^2 + (10-x)^2$$
$$= x^2 + 100 - 20x + x^2$$
$$= 2x^2 - 20x + 100$$

$S'(x) = 4x - 20 \Rightarrow$ c. pt. at $x = 5$

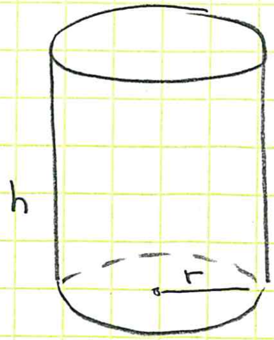
$-$ $+$

x	0	5	10
$S'(x)$	$- \; - \; - \; -$	0	$+ \; + \; + \; +$
$S(x)$	100 $\searrow$	50	$\nearrow$ 100

$S(0) = 100$
$S(10) = 100$
$S(5) = 50$

$\Rightarrow$ (a) S is <u>minimal</u> at $x=5, y=5$; (b) S is <u>maximal</u> at $\begin{cases} x=0, y=10 \\ x=10, y=0 \end{cases}$

(13) A cylindrical can (w/a top lid) must contain 300 cm^3 of liquid. Dimensions that will minimize the cost of metal needed to make can?



Surface area: $2\pi r h + 2\pi r^2$ ← minimize

Volume: $\pi r^2 h = 300$ (constraint)

$h = \dfrac{300}{\pi r^2}$ (replace in Surface Area)

$$S = 2\pi r h + 2\pi r^2 = 2\cancel{\pi}\cancel{r} \cdot \frac{300}{\cancel{\pi} r^{\cancel{2}}} + 2\pi r^2$$

$$\Rightarrow S(r) = \frac{600}{r} + 2\pi r^2$$

$$S'(r) = \frac{-600}{r^2} + 4\pi r$$

$$S'(r) = 0 \Rightarrow 4\pi r = \frac{600}{r^2}$$

$$4\pi r^3 = 600 \Rightarrow r^3 = \frac{600}{4\pi} = \frac{150}{\pi}$$

$$\Rightarrow r = \sqrt[3]{\frac{150}{\pi}}$$

$$\Rightarrow h = \frac{300}{\pi\left(\sqrt[3]{\frac{150}{\pi}}\right)^2}$$

Justification: $S'(r) = \dfrac{-600}{r^2} + 4\pi r$

$$= \frac{4\pi r^3 - 600}{r^2}$$

r	0	$\sqrt[3]{\frac{150}{\pi}}$	
$S'(r)$	$- - -$	0	$+ + + +$
$S(r)$	↘	min	↗

14 300 cm^2 of material; cylindrical can w/ lid; max volume?

Surface area $= 2\pi r h + 2\pi r^2 = 300$ ← constraint

Volume $= \pi r^2 h$ ← maximize

$$2\pi r h + 2\pi r^2 = 300 \quad \Big/ \frac{1}{2\pi}$$

$$rh + r^2 = \frac{150}{\pi}\ ; \quad rh = \frac{150}{\pi} - r^2 ; \quad h = \frac{150}{\pi}\frac{1}{r} - r$$

$$\text{Vol} = V(r) = \pi r^2 h = \pi r^2\left(\frac{150}{\pi}\frac{1}{r} - r\right) = \underline{\underline{150r - \pi r^3}} = V(r)$$

$$V'(r) = 150 - 3\pi r^2$$

$$V'(r) = 0 \Rightarrow 3\pi r^2 = 150\ ; \ r^2 = \frac{50}{\pi}$$

$$\Rightarrow r = \sqrt{\frac{50}{\pi}}$$

$$\Downarrow$$

$$h = \frac{150}{\pi}\sqrt{\frac{\pi}{50}} - \sqrt{\frac{50}{\pi}}$$

Justification:

r	$-\sqrt{\frac{50}{\pi}}$	0	$\sqrt{\frac{50}{\pi}}$	
$V'(r)$	$-$ 0 $+$	$+$ $+$	0	$-$ $-$ $-$
$V(r)$		↗	max	↘

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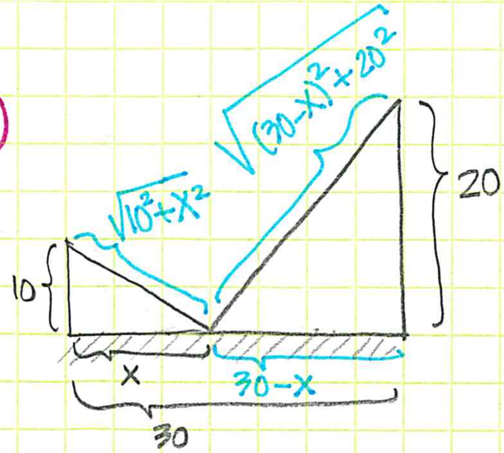


Amount of wire needed:

$$\sqrt{100+x^2} + \sqrt{(30-x)^2+400} = L(x)$$

minimize

$$L'(x) = \frac{2x}{2\sqrt{100+x^2}} + \frac{-(30-x)\cdot 2}{2\sqrt{(30-x)^2+400}}$$

$$= \frac{x}{\sqrt{100+x^2}} - \frac{30-x}{\sqrt{(30-x)^2+400}}$$

$$L'(x)=0 \Rightarrow \frac{x}{\sqrt{100+x^2}} = \frac{30-x}{\sqrt{(30-x)^2+400}}$$

$$\Rightarrow x\sqrt{(30-x)^2+400} = (30-x)\sqrt{100+x^2} \quad \Big|^{\wedge 2}$$

$$\Rightarrow x^2\left((30-x)^2+400\right) = (30-x)^2(100+x^2)$$

$$\cancel{x^2(30-x)^2} + 400x^2 = 100(30-x)^2 + \cancel{x^2(30-x)^2}$$

$$400x^2 = 100(30-x)^2$$

$$4x^2 = (30-x)^2$$

$$4x^2 = 900 - 60x + x^2$$

$$3x^2 + 60x - 900 = 0$$

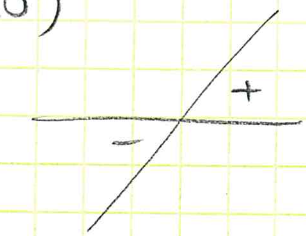
$$x^2 + 20x - 300 = 0$$

$$(x+30)(x-10) = 0 \Rightarrow \cancel{x=-30} \text{ or } \underline{x=10}$$

$\Rightarrow$ The wire must be positioned 10 ft from the first pole.

(16) Minimize $(x-70)^2+(x-80)^2+(x-120)^2$?

$$f'(x) = 2(x-70)+2(x-80)+2(x-120)$$
$$= 2\left(x-70+x-80+x-120\right)$$
$$= 2(3x-270)$$



$$f'(x)=0 \Rightarrow 3x=270 \Rightarrow x=90$$

x		90	
$f'(x)$	$-\;-\;-$	0	$+\;+\;+\;+$
$f(x)$	$\infty \searrow$	min	$\nearrow \infty$