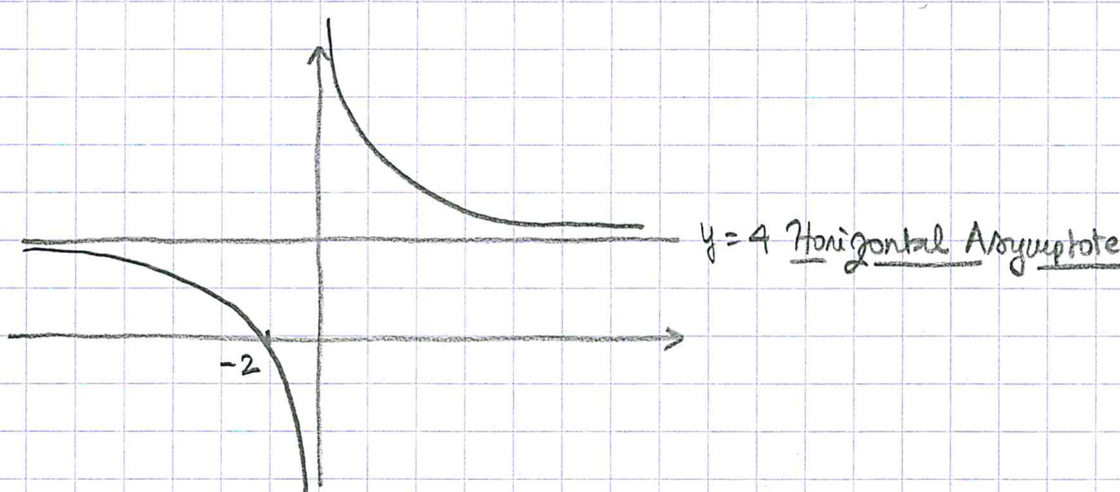
Worksheet 3
Exam 2

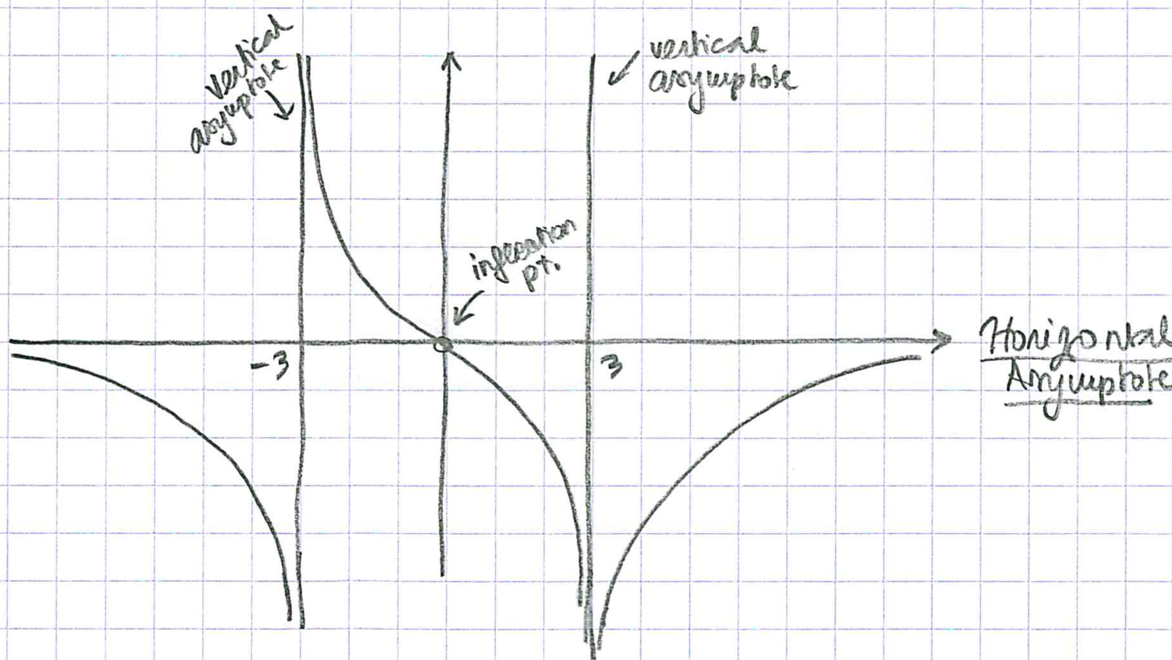
1.

x	$-\infty$ → -2 → 0	0 → $+\infty$
$f'(x)$	− − − − − −	− − − − − − − −
$f''(x)$	− − − − − − −	+ + + + + +
$f(x)$	4 ⌢↘ 0 ↘⌢ $-\infty$	$+\infty$ ↘ ∪ 4

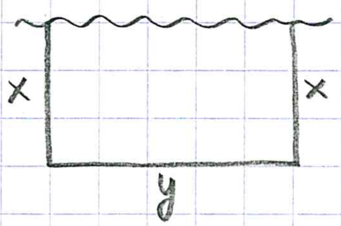


2.

x	$-\infty$ → -3	-3 → 0 → 3	3 → ∞
$f'(x)$	− − − −	− − − −	+ + + +
$f''(x)$	− − − −	+ + + 0 − −	− − − −
$f(x)$	0 ↘⌢ $-\infty$	$+\infty$ ∪ ↘ 0 ↘⌢ $-\infty$	$-\infty$ ↗⌢ 0



③ 2400 ft of fencing



$2x + y = 2400 \Rightarrow y = 2400 - 2x$

$\text{Area} = xy = x(2400 - 2x) = 2400x - 2x^2 = A($

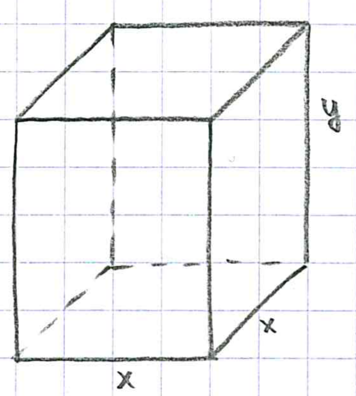
$A'(x) = 2400 - 4x$

$A'(x) = 0 \Rightarrow x = \frac{2400}{4} = 600$

$\Rightarrow y = 2400 - 1200 = 1200$

x		600	
A'(x)	+ +	0	− − −
A(x)	↗	max	↘

④



$10\,m^2 = (xy)\cdot 4 + (x^2)\cdot 2$

$10 = 4xy + 2x^2$

$\frac{10 - 2x^2}{4x} = y \Rightarrow y = \frac{5 - x^2}{2x}$

$\text{Vol} = (x^2)\cdot y = x^2 \cdot \frac{5 - x^2}{2x} = \frac{x(5 - x^2)}{2} = V(x)$

$V(x) = \frac{1}{2}(5x - x^3)$

$V'(x) = \frac{1}{2}(5 - 3x^2)$

$x = \pm\sqrt{\frac{5}{3}}, \quad x > 0 \Rightarrow x = \sqrt{\frac{5}{3}}$

x		$-\sqrt{\frac{5}{3}}$		0		$\sqrt{\frac{5}{3}}$	
V'(x)	−	0	+		+	0	− −
V(x)					↗	max	↘

$\Rightarrow V_{max} = V\left(\sqrt{\frac{5}{3}}\right) = \frac{1}{2}\left(5\cdot\sqrt{\frac{5}{3}} - \frac{5}{3}\sqrt{\frac{5}{3}}\right)$

$= \frac{1}{2}\sqrt{\frac{5}{3}}\left(5 - \frac{5}{3}\right) = \frac{5}{3}\sqrt{\frac{5}{3}}$

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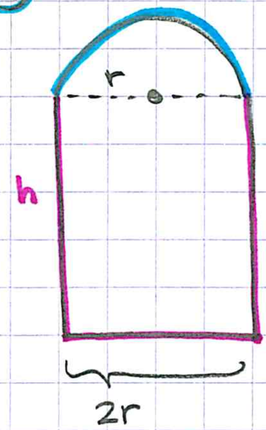


Outside Perimeter = 9

$$9 = (2h+2r) + (\pi r)$$ ← length of semicircle

$$2h = 9 - 2r - \pi r$$
$$= 9 - (2+\pi)r$$
$$\boxed{h = \frac{9}{2} - \frac{(2+\pi)}{2} r}$$

$$\text{Area} = (h \cdot 2r) + \frac{1}{2}(\pi r^2) = 2r\left(\frac{9}{2} - \frac{(2+\pi)}{2} r\right) + \frac{\pi}{2} r^2$$
$$= 9r - (2+\pi)r^2 + \frac{\pi}{2} r^2$$
$$= 9r - \left(2+\pi-\frac{\pi}{2}\right) r^2$$
$$= \boxed{9r - \left(2+\frac{\pi}{2}\right) r^2}$$

$$A = r(9-2r-\pi r) + \frac{1}{2}\pi r^2$$

ⓑ Maximal area?

$$A(r) = 9r - \left(2+\frac{\pi}{2}\right) r^2$$

$$A'(r) = 9 - \left(2+\frac{\pi}{2}\right) \cdot 2r = 9 - (4+\pi) r$$

$$A'(r) = 0 \Rightarrow r = \frac{9}{4+\pi}$$

r	0		$\frac{9}{4+\pi}$	
$A'(r)$		$+$	0	$- -$
		$\nearrow$	max	$\searrow$

$$\Rightarrow A_{max} = A\left(\frac{9}{4+\pi}\right) = 9 \cdot \frac{9}{4+\pi} - \left(2+\frac{\pi}{2}\right)\left(\frac{9}{4+\pi}\right)^2$$

$$= \frac{81}{4+\pi} - \frac{\pi+4}{2} \cdot \frac{81}{(\pi+4)^2} = \frac{81}{4+\pi} - \frac{1}{2} \cdot \frac{81}{\pi+4}$$

$$= \frac{81}{2(\pi+4)}$$

(6) Point on $2x+y+3=0$ closest to the point $(-2,-3)$.

$$\left.\begin{array}{l}(\text{dist})^2 = (x+2)^2+(y+3)^2 \\ y=-2x-3\end{array}\right\} \Rightarrow d(x)=(x+2)^2+(-2x-3+3)^2$$

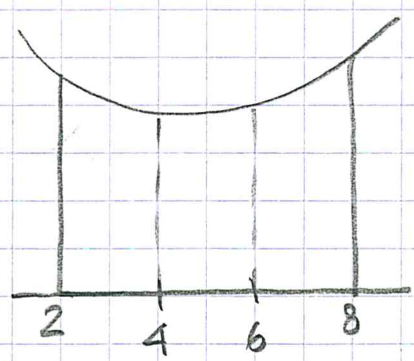
$$= (x+2)^2+(-2x)^2$$
$$= x^2+4x+4+4x^2$$
$$= 5x^2+4x+4 \quad \leftarrow \underline{\text{minimize}}$$

$$d'(x) = 10x+4 \Rightarrow x = -\frac{2}{5} \Rightarrow y = -2\left(-\frac{2}{5}\right)-3 = \frac{4}{5}-3 = -\frac{11}{5}$$

x			$-2/5$		
$d'(x)$	$-$	$-$	0	$+$	$+$
$d(x)$	$\searrow$		min	$\nearrow$	

$\Rightarrow \left(-\frac{2}{5}, -\frac{11}{5}\right)$

(7) Estimate the area under $f(x)=x^2+3x$ from $x=2$ to $x=8$ using the areas of 3 rectangles of equal width, w/ left/right endp:



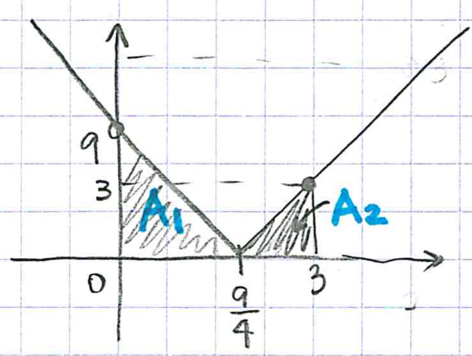
$$\frac{8-2}{3} = \frac{6}{3} = 2$$

$$L_3 = 2\cdot f(2) + 2\cdot f(4) + 2\cdot f(6)$$
$$= 2\cdot(4+6) + 2\cdot(16+12) + 2\cdot(36+18)$$

$$R_3 = 2\cdot f(4) + 2\cdot f(6) + 2\cdot f(8)$$
$$= 2\cdot(16+12) + 2\cdot(36+18) + 2\cdot(64+24)$$

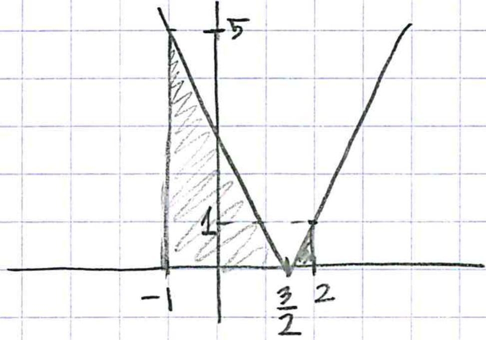
(8) $\int_0^3 |4x-9|\,dx$ $= A_1 + A_2 = \frac{1}{2}\left(\frac{9}{4}\cdot 9\right) + \frac{1}{2}\left(3-\frac{9}{4}\right)\cdot(3)$

$$= \frac{1}{2}\cdot\frac{81}{4} + \frac{3}{2}\cdot\frac{3}{4} = \frac{81+9}{8} = \frac{90}{8}$$



(9) $\int_{-1}^{2} |2x-3|\,dx$ $= \frac{1}{2}\left(5\cdot\left(1+\frac{3}{2}\right)\right) + \frac{1}{2}\left(2-\frac{3}{2}\right)\cdot 1$

$= \frac{5}{2}\cdot\frac{5}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{26}{4}$

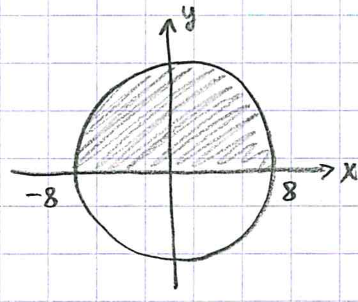


(10) $\int_{-8}^{8} \sqrt{64-x^2}\,dx = ?$ $= \frac{1}{2}\left(\pi\cdot 8^2\right) = 32\pi$

$y = \sqrt{64-x^2} \geq 0$

$y^2 = 64 - x^2$

$x^2 + y^2 = 64$



(11) $g(x) = \int_{1}^{x} \frac{1}{1+t^4}\,dt \Rightarrow g'(x) = \frac{1}{1+x^4}$ by FTC I.

(12) $\int_{0}^{6} (10x^2 - 6x + 2)\,dx = \left(10\frac{x^3}{3} - 6\frac{x^2}{2} + 2x\right)\Big|_0^6 = 10\cdot\frac{6^3}{3} - 6\cdot\frac{6^2}{2} + 2\cdot 6$

(13) $\int_{-\pi/4}^{\pi/3} \left(\sec^2(x) + \sin(x)\right)dx = \left(\tan(x) - \cos(x)\right)\Big|_{-\pi/4}^{\pi/3}$

$= \left(\tan\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{3}\right)\right) - \left(\tan\left(-\frac{\pi}{4}\right) - \cos\left(-\frac{\pi}{4}\right)\right)$

$= \left(\sqrt{3} - \frac{1}{2}\right) - \left(-1 - \frac{\sqrt{2}}{2}\right) = \sqrt{3} - \frac{1}{2} + 1 + \frac{\sqrt{2}}{2} = \sqrt{3} + 1 + \frac{\sqrt{2}-1}{2}$

(14) $\int_{1}^{\sqrt{7}} \frac{15s^5 + 3\sqrt{s}}{s^5}\,ds = \int_{1}^{\sqrt{7}} \left(15 + 3s^{1/2-5}\right)ds = \int_{1}^{\sqrt{7}} \left(15 + 3s^{-9/2}\right)ds$

$= \left(15s + 3\frac{s^{-7/2}}{-7/2}\right)\Big|_1^{\sqrt{7}} = \left(15s - \frac{6}{7s^{7/2}}\right)\Big|_1^{\sqrt{7}}$

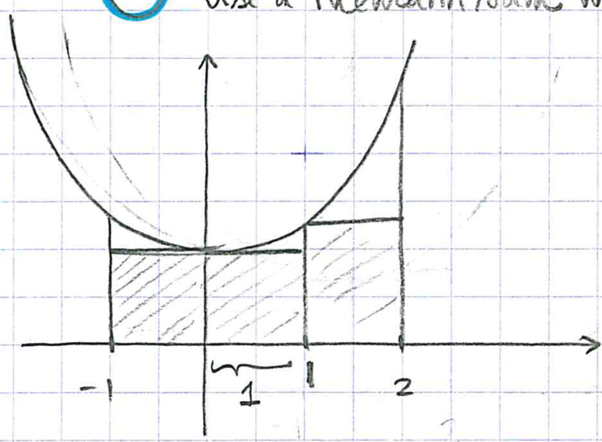
$= \left(15\sqrt{7} - \frac{6}{7(7)^{7/4}}\right) - \left(15 - \frac{6}{7}\right)$

15. $$\int_3^1 \left(\frac{3}{x^2}-7\right)dx = -\int_1^3 \left(3x^{-2}-7\right)dx$$

$$= -\left(3\frac{x^{-1}}{-1}-7x\right)\Bigg|_1^3 = -\left(-\frac{3}{x}-7x\right)\Bigg|_1^3 = \left(\frac{3}{x}+7x\right)\Bigg|_1^3$$

$$= \left(\frac{3}{3}+7\cdot 3\right)-\left(\frac{3}{1}+7\cdot 1\right) = 22-10 = 12$$

16. Use a Riemann sum with 3 rectangles to <u>underestimate</u> the area under $y=x^2+3$ between $x=-1$ and $x=2$.



$$A = 1\cdot f(0)+1\cdot f(0)+1\cdot f(1)$$

$$= 1\cdot 3+1\cdot 3+1\cdot 4 = 10$$

17. Express as a sigma notation:

$$2+6+12+20+30$$

(a) $\displaystyle\sum_{k=1}^{5}(k^2-k)$ $= 0+\ldots$

(c) $\displaystyle\sum_{k=0}^{4}(2^k-k)$ $= 1+\ldots$

(b) $\displaystyle\sum_{k=0}^{4}(k^2+k)$ $= \cancel{0}+\ldots$

(d) $\displaystyle\sum_{k=1}^{5}(k^2+k) = 2+6+12+20+30$ ✓