Exam 2 - Worksheet 2

① $\lim\limits_{x\to\infty} \dfrac{2x^3-11x^2-5x}{2-7x-5x^3}$

divide EVERY term by the highest power of x in the denominator (here x^3)

$$= \lim_{x\to\infty} \frac{2-\underbrace{11\cdot\frac{1}{x}}_{\to 0}-\underbrace{5\cdot\frac{1}{x^2}}_{\to 0}}{\underbrace{\frac{2}{x^3}}_{\to 0}-\underbrace{\frac{7}{x^2}}_{\to 0}-5} = -\frac{2}{5}$$

② $\lim\limits_{x\to\infty} \left(\sqrt{4x^2+3x}-2x\right) =$ multiply by the conjugate

$$= \lim_{x\to\infty} \frac{\left(\sqrt{4x^2+3x}-2x\right)\cdot\left(\sqrt{4x^2+3x}+2x\right)}{\sqrt{4x^2+3x}+2x}$$

$$= \lim_{x\to\infty} \frac{4x^2+3x-(2x)^2}{\sqrt{4x^2+3x}+2x} = \lim_{x\to\infty} \frac{3x}{\sqrt{4x^2+3x}+2x}$$

divide EVERY term by x

$$= \lim_{x\to\infty} \frac{\frac{3x}{x}}{\frac{\sqrt{4x^2+3x}}{x}+\frac{2x}{x}} = \lim_{x\to\infty} \frac{3}{\sqrt{\frac{4x^2+3x}{x^2}}+2}$$

$$= \lim_{x\to\infty} \frac{3}{\sqrt{4+\underbrace{\frac{3}{x}}_{\to 0}}+2} = \frac{3}{\sqrt{4}+2} = \frac{3}{4}$$

③ $\lim\limits_{x\to\infty} \left(\sqrt{2x+3}-2x\right) = \lim\limits_{x\to\infty} \dfrac{2x+3-4x^2}{\sqrt{2x+3}+2x}$

divide every term by x

$$= \lim_{x\to\infty} \frac{2+\frac{3}{x}-4x}{\sqrt{\frac{2x+3}{x^2}}+2} = \lim_{x\to\infty} \frac{2+\overbrace{\frac{3}{x}}^{\to 0}\overbrace{-4x}^{\to -\infty}}{\sqrt{\underbrace{\frac{2}{x}}_{\to 0}+\underbrace{\frac{3}{x^2}}_{\to 0}}+2} = \frac{2-\infty}{2} = -\infty$$

4. $$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)=\lim_{x\to\infty}\frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}$$

$$=\lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}=\lim_{x\to\infty}\frac{1}{\sqrt{9+\frac{1}{x}}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{6}$$

$\frac{1}{x}\to 0$

5. $$\lim_{x\to\infty}\left(\sqrt{x+1}-3x\right)=\lim_{x\to\infty}\frac{x+1-9x^2}{\sqrt{x+1}+3x}=\lim_{x\to\infty}\frac{1+\frac{1}{x}-9x}{\sqrt{1+\frac{1}{x}}+3}=-\infty$$

6. $$\lim_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)=\lim_{x\to\infty}\frac{\cancel{x}+1-\cancel{x}}{\sqrt{x+1}+\sqrt{x}}=\lim_{x\to\infty}\frac{1}{\sqrt{x+1}+\sqrt{x}}=0.$$

7. Which of the functions has a horizontal asymptote: ?

a) $f(x)=\frac{x}{x^2+1}$; b). $f(x)=\frac{3x^3+9}{2x^3+x^2+1}$; c). $f(x)=\frac{x^5}{x^3+1}$

Degrees must be <u>the same</u> in order to have a horizontal asymptote.

Horizontal asymptote at y=0

8. Which has a vertical asymptote?

a). $f(x)=\frac{x}{x^2+1}$; b). $f(x)=\frac{x^2}{x^2+1}$; c). $f(x)=\frac{x}{x^2-1}$

Denominator must be 0 to have a vertical asymptote.

9. Which has a slant asymptote?

a). $f(x)=\frac{2x^2-1}{x+1}$; b). $f(x)=\frac{2x^2-1}{2x^2+1}$; c). $f(x)=\frac{x^3}{x^4+1}$

Deg (num.) = Deg (denom.) + 1 to have slant asymptotes.

(10) $\int (16x^3 - 21x^2 + 12x - 10)\,dx = 16 \cdot \frac{x^4}{4} - 21\frac{x^3}{3} + 12\frac{x^2}{2} - 10x + C$

$= \boxed{4x^4 - 7x^3 + 6x^2 - 10x + C}$

(11) $\int (36x^3 - 21x^2 + 14x - 4)\,dx = 36\frac{x^4}{4} - 21\frac{x^3}{3} + 14\frac{x^2}{2} - 4x + C$

$= \boxed{9x^4 - 7x^3 + 7x^2 - 4x + C}$

(12) $\int (4x^9 + 5\sec x \tan x)\,dx = \boxed{4 \cdot \frac{x^{10}}{10} + 5\sec x + C}$

(13) $\int \frac{7 - 6x^7}{x^4}\,dx = \int \left(\frac{7}{x^4} - \frac{6x^7}{x^4}\right)dx = \int (7x^{-4} - 6x^3)\,dx$

$= \boxed{7\frac{x^{-3}}{-3} - 6\frac{x^4}{4} + C}$

(14) $\int \frac{5 - 3x^8}{x^4}\,dx = \int \left(\frac{5}{x^4} - \frac{3x^8}{x^4}\right)dx = \int (5x^{-4} - 3x^4)\,dx$

$= \boxed{5\frac{x^{-3}}{-3} - 3\frac{x^5}{5} + C}$

(15) $\int \left(\frac{7}{\sqrt[3]{x}} - 6\sqrt[3]{x^2}\right)dx = \int \left(7x^{-1/3} - 6x^{2/3}\right)dx = \boxed{7\frac{x^{2/3}}{2/3} - 6\frac{x^{5/3}}{5/3} + C}$

(16) $\int \left(4\sqrt[5]{x^7} - \frac{2}{\sqrt[6]{x}}\right)dx = \int \left(4x^{7/5} - 2x^{-1/6}\right)dx = \boxed{4\frac{x^{12/5}}{12/5} - 2\frac{x^{5/6}}{5/6} + C}$

(17) $a(t) = (t+2)^3$

(a) $v(0) = 7$; $v(t) = ?$ $\qquad v(t) = \int (t+2)^3\,dt = \frac{(t+2)^4}{4} + C$

$\boxed{v(t) = \frac{1}{4}(t+2)^4 + 3}$ $\qquad v(0) = \frac{2^4}{4} + C = 7 \Rightarrow C = 7 - 4 \Rightarrow C = 3$

(b) $s(0) = 5$; $s(t) = ?$ $\qquad s(t) = \int \left(\frac{1}{4}(t+2)^4 + 3\right)dt = \frac{1}{4}\frac{(t+2)^5}{5} + 3t + C$

$s(0) = \frac{2^5}{20} + C = 5 \Rightarrow C = 5 - \frac{8}{5} = \frac{17}{5}$

$\boxed{s(t) = \frac{(t+2)^5}{20} + 3t + \frac{17}{5}}$

18 $\int (4\sec^2 x - 4)\,dx = 4\tan(x) - 4x + C$

19 $\int (4\sin x - 3\cos x)\,dx = -4\cos(x) - 3\sin(x) + C$

20 $\int (-\pi \sin x + \sec(x)\tan(x))\,dx = +\pi\cos(x) + \sec(x) + C.$

21 $\int (9\cos x - \sqrt[10]{x^3})\,dx = 9\sin x - \dfrac{x^{13/10}}{13/10} + C$

22 $\sum_{i=1}^{n} (8i+6) = 8\left(\sum_{i=1}^{n} i\right) + \left(\sum_{i=1}^{n} 6\right) = 8 \cdot \dfrac{n(n+1)}{2} + 6n.$

23 $\sum_{i=1}^{n} (9i^2+8i) = 9\left(\sum_{i=1}^{n} i^2\right) + 8\left(\sum_{i=1}^{n} i\right) = 9\,\dfrac{n(n+1)(2n+1)}{6} + 8\,\dfrac{n(n+1)}{2}$

24 $\sum_{i=1}^{10} (2i+4) = 2\left(\sum_{i=1}^{10} i\right) + \left(\sum_{i=1}^{10} 4\right) = \cancel{2} \cdot \dfrac{10 \cdot 11}{\cancel{2}} + 10 \cdot 4 = 110 + 40 = 150.$

25 $\sum_{i=1}^{10} (6i^2+2i) = 6\left(\sum_{i=1}^{10} i^2\right) + 2\left(\sum_{i=1}^{n} i\right) = \cancel{6} \cdot \dfrac{10 \cdot 11 \cdot 21}{\cancel{6}} + \cancel{2} \cdot \dfrac{10 \cdot 11}{\cancel{2}} = 2310 + 110$
$= 2420.$