Exam 2 Review: Worksheet 1

1. [Multiple Choice] Use a linear approximation to estimate $\sqrt{17}$:

(A) $4 + \sqrt{17}$

(B) $4 + 1/2$

(C) $4 + 1/4$

(D) $4 + 1/8$

(E) $4 + 1/16$

2. [Standard Response] Use a linear approximation to estimate $\sin(32°)$.

3. Suppose the radius of a sphere was measured to be 5cm, with a possible error of 1/5cm. Use differentials to estimate:

a). The maximum error in the calculated surface area?

b). The maximum error in the calculated volume?

4. Suppose the edge of a cube was measured to be 30cm, with a possible error in measurement of 0.1cm. Use differentials to estimate:

a). The maximum error in the calculated volume?

b). The maximum error in the calculated surface area?

5. Find the critical numbers for the functions:

a). $f(x) = x^2 - 2x + 4$

b). $f(x) = x^3 - \frac{9}{2}x^2 - 54x + 2$

c). $f(x) = 4x - \sqrt{x^2 + 1}$

d). $f(x) = \frac{x}{x^2+1}$

e). $f(x) = 4x^{2/3} - 5x^{5/3}$

f). $f(x) = x^{1/9} - x^{-8/9}$

For problems 6 – 11, find the **absolute** minimum and maximum of the given function on the given interval:
Make sure to review the method of finding absolute minima and maxima of continuous functions **on a closed interval** *(see notes for 3.1).*

6. $f(x) = x^3 - 3x^2 + 1;\ \ x \in [1, 4]$.

7. $f(x) = x - 2\sin(x);\ \ x \in [0, \pi]$.

8. $f(x) = 3x^2 - 12x + 5;\ \ x \in [0, 3]$.

9. $f(x) = x^4 - 4x^2 + 2;\ \ x \in [-1, 2]$.

10. $f(x) = \dfrac{x}{x^2 + 1};\ \ x \in [0, 2]$.

11. $f(x) = \sin x + \cos x;\ \ x \in [0, \pi/3]$.

For problems 12 – 14, find all the following for the given functions:

a). Find all critical numbers and classify each one as a local max, a local min, or neither.

b). Find the intervals where f is increasing and decreasing.

c). Find the intervals where f is concave up, and concave down.

d). Find any inflection points.

12. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

13. $f(x) = x^3 - 12x + 1$

14. $f(x) = x(x - 8\sqrt{x})$

15. If $f(x) = x^2 - 3x$, which of the following statements is true by the Mean Value Theorem?

(A) There is a value c in the interval $(0, 4)$ such that $f(c) = 1$.

(B) There is a value c in the interval $(0, 4)$ such that $f'(c) = 1$.

(C) There is a value c in the interval $(0, 4)$ such that $f(c) = 4$.

(D) There is a value c in the interval $(0, 4)$ such that $f'(c) = 4$.

(E) The Mean Value Theorem cannot be applied.

16. Find a positive value c for x which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = 3x^2 - 5x + 1$ on the interval $[2, 5]$.

(A) 1/2

(B) 13/6

(C) 10/6

(D) 7/2

(E) 2/3

17. What can one conclude by applying the Mean Value Theorem to the function $f(x) = \frac{1}{x} - 2$ on the interval $[1, 3]$?

(A) The graph of f has a tangent line between $x = 1$ and $x = 3$ with slope $1/3$.

(B) The graph of f has a tangent line between $x = 1$ and $x = 3$ with slope $-1/3$.

(C) The function has a zero in the interval $[1, 3]$.

(D) The MVT does not apply because f is not continuous on $[1, 3]$.

(E) The MVT does not apply because f is not differentiable on $(1, 3)$.

18. Apply Newton's Method to approximate the root of the equation $x^3 - 2x^2 - 1 = 0$. If we start at $x_1 = 2$, then after one iteration of the method, x_2 is:

(A) 1/4

(B) 1/9

(C) 9/4

(D) 3/4

(E) 2/3