1. Use a linear approx. to estimate $\sqrt{17}$:

   (a) $4+\sqrt{17}$
   (b) $4+1/2$
   (c) $4+1/4$
   → (d) $4+1/8$
   (e) $4+1/16$

Linear approx. of f at $x=a$:

$$f(x) \approx f(a)+f'(a)(x-a)$$

$f(x)=\sqrt{x}$; use $a=16$, $x=17$

$$f'(x)=\frac{1}{2\sqrt{x}}$$

$$\sqrt{17} \approx f(16)+f'(16)(17-16) = 4+\frac{1}{2\sqrt{16}}(17-6) = 4+\frac{1}{8}$$

2. Use a linear approx. to estimate $\sin(32°)$

$f(x)=\sin(x)$; Take $x=32°$, $a=30°=\frac{\pi}{6}$

$f'(x)=\cos(x)$

$$\sin(32°) \approx \sin\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{6}\right)\left(\frac{32\pi}{180}-\frac{\pi}{6}\right)$$

$\frac{30\pi}{180}$

$180 — \pi$
$32 — ?$

$32° = \frac{32\pi}{180}$

$$\approx \frac{1}{2}+\frac{\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}\pi}{180} = \frac{1}{2}+\frac{\pi\sqrt{3}}{180}$$

3. Use differentials to solve: the radius of a sphere was measured to be 5cm w/ a possible error of $\frac{1}{5}$cm.

$y=f(x)$
$dy=f'(x)dx$
change in y — change in x

   (a) Max error in calculated surface area?

$S=4\pi R^2$

$dS=4\pi\cdot 2R\,dR$

↳ error in R: $\Delta R \approx dR = 1/5$

$$dS=4\pi\cdot 2\cdot \cancel{5}\cdot\frac{1}{\cancel{5}} = 8\pi$$

   (b) Max error in calculated volume?

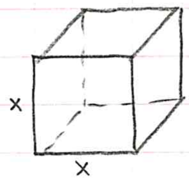
$V=\frac{4}{3}\pi R^3$

$dV=\frac{4}{3}\pi\cdot 3R^2 dR$

$$dV=\frac{4}{\cancel{3}}\pi\cdot\cancel{3}\cdot 5^{\cancel{2}}\cdot\frac{1}{\cancel{5}} = 20\pi$$

4) The edge of a cube was found to be 30 cm w/ a possible error in measurement of 0.1 cm. Use differentials to estimate the max possible error when computing:

a) Volume of the cube? Denote the side length of the cube by x.



$$V = x^3$$

$$dV = 3x^2\,dx$$

$$\approx 3\cdot(30)^2\cdot(0.1) = 3\cdot 900\cdot \frac{1}{10} = 270 \ (\text{cm}^3)$$

b) Surface area of cube?

$$S = x^2\cdot 6$$

$$dS = 2x\cdot 6\,dx$$

$$\approx 2(30)\cdot 6\cdot(0.1) = 12\cdot 30\cdot\frac{1}{10} = 36 \ (\text{cm}^2)$$

5) Find the critical points:

a) $f(x) = x^2 - 2x + 4$

$f'(x) = 2x - 2$ $\quad x = 1$

b) $f(x) = x^3 - \frac{9}{2}x^2 - 54x + 2$

$$f'(x) = 3x^2 - \frac{9}{2}\cdot 2x - 54 = 3x^2 - 9x - 54$$

$$= 3(x^2 - 3x - 18) = 3(x-6)(x+3)$$

$$x = -3, 6$$

c) $f(x) = 4x - \sqrt{x^2+1}$

$$f'(x) = 4 - \frac{2x}{2\sqrt{x^2+1}} = 4 - \frac{x}{\sqrt{x^2+1}}$$

$$4 = \frac{x}{\sqrt{x^2+1}}$$

$$4\sqrt{x^2+1} = x$$

$$16(x^2+1) = x^2$$

$$16x^2 + 16 = x^2$$

$$15x^2 = -16$$

$$x^2 = -\frac{16}{15} \quad \text{(not possible)}$$

No critical numbers.

(d) $f(x) = \dfrac{x}{x^2+1}$

$$f'(x) = \frac{(x^2+1) - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$f'(x) = 0 \iff 1 - x^2 = 0 \iff x = \pm 1$

(e) $f(x) = 4x^{2/3} - 5x^{5/3}$

$$f'(x) = 4 \cdot \frac{2}{3} x^{-1/3} - 5 \cdot \frac{5}{3} x^{2/3} = \frac{8}{3x^{1/3}} - \frac{25x^{2/3}}{3} = \frac{8-25x}{3x^{1/3}}$$

$f'(x) = 0 \iff 8 - 25x = 0 \iff x = 8/25$

$f'(x)$ dne at $x=0$ (and 0 is in the domain)

$\Rightarrow$ C.#'s: $\frac{8}{25}, 0$

(f) $f(x) = x^{1/9} - x^{-8/9}$

$$f'(x) = \frac{1}{9} x^{-8/9} + \frac{8}{9} x^{-17/9} = \frac{1}{9x^{8/9}} + \frac{8}{9x^{17/9}} = \frac{x+8}{9x^{17/9}}$$

$\Rightarrow f'(x) = 0$ when $x = -8$ and $f'(x)$ dne at $x = 0$

but 0 is not in the domain of f $\Rightarrow$ C.#'s: -8

(6) Find the absolute min & max of $f(x) = x^3 - 3x^2 + 1$ on $x \in [1,4]$

Critical numbers: $f'(x) = 3x^2 - 6x = 3x(x-2) \Rightarrow x=0, x=2$

only $x=2$ is in $[1,4]$

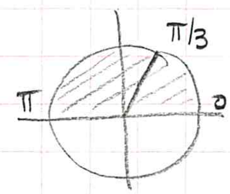
Evaluate at critical numbers in the interval: $f(2) = 8 - 12 + 1 = -3 \leftarrow$ **MIN**

Evaluate at endpoints: $f(1) = 1 - 3 + 1 = -1$

$f(4) = 64 - 48 + 1 = 17 \leftarrow$ **MAX**

$\Rightarrow$ The absolute max occurs at $(4, 17)$ and the absolute min at $(2, -3)$.

(7) $f(x) = x - 2\sin x, \quad 0 \leq x \leq \pi$



Critical numbers: $f'(x) = 1 - 2\cos x$

$f'(x) = 0 \Leftrightarrow 1 - 2\cos x = 0 \Leftrightarrow \cos x = \frac{1}{2}$

$\Leftrightarrow x = \pi/3$ (between 0 & π)

Evaluate at c. #'s: $f(\pi/3) = \frac{\pi}{3} - 2 \cdot \sin\frac{\pi}{3} = \frac{\pi}{3} - 2 \cdot \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \sqrt{3} < 0$

MIN

Evaluate at endpts: $f(0) = 0 - 2\sin(0) = 0$

$f(\pi) = \pi - 2\sin(\pi) = \pi \leftarrow$ **MAX**

$\frac{\pi}{3} - \sqrt{3} = \frac{\pi - 3\sqrt{3}}{3} < 0$ because $\pi \approx 3.14$ and $\sqrt{3}$ is "close" to 2

$\Rightarrow$ Absolute max @ (π, π); Absolute min @ $(\pi/3, \pi/3 - \sqrt{3})$.

8) $f(x) = 3x^2 - 12x + 5; \quad x \in [0,3].$

Critical no.'s: $f'(x) = 6x - 12 \Rightarrow x = 2$ (in interval ✓)

Evaluate: $f(2) = 3 \cdot 4 - 24 + 5 = -7$ ← MIN

$f(0) = 5$ ← MAX

$f(3) = 3 \cdot 9 - 36 + 5 = -4$

Absolute max: $(0,5)$. Absolute min: $(2,-7)$.

9) $f(x) = x^4 - 4x^2 + 2, \quad x \in [-1,2]$

Critical no.'s: $f'(x) = 4x^3 - 8x = 4x(x^2-2) \Rightarrow 0, -\sqrt{2}, \sqrt{2}$

Out of these, only $0, \sqrt{2}$ are in $[-1,2]$

Evaluate: $f(0) = 2$ ← MAX

$f(\sqrt{2}) = 4 - 4 \cdot 2 + 2 = -2$ ← MIN

$f(-1) = 1 - 4 + 2 = -1$

$f(2) = 16 - 16 + 2 = 2$ ← MAX

Absolute max: at $(2,2)$ & $(0,2)$. Absolute min: $(\sqrt{2}, -2)$.

10) $f(x) = \dfrac{x}{x^2+1}, \quad x \in [0,2]$

Critical no.'s: $f'(x) = \dfrac{(x^2+1) - x \cdot 2x}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2} \Rightarrow \pm 1$ but only $1 \in [0,2]$

Evaluate: $f(0) = 0$ ← MIN

$f(1) = \frac{1}{2}$ ← MAX

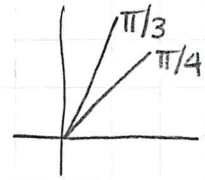
$f(2) = \frac{2}{5}$

$\frac{1}{2} \boxed{>} \frac{2}{5}$ 1×10

$5 \boxed{>} 4$

Absolute max at $(1, 1/2)$; Absolute min: $(0,0)$.

(11) $f(x) = \sin x + \cos x, \quad x \in [0, \pi/3].$



$f'(x) = \cos x - \sin x \Rightarrow$ critical # at $x = \pi/4$

$f(0) = 1 \leftarrow$ min

$f(\pi/3) = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

$f(\pi/4) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \leftarrow$ max

$\frac{1+\sqrt{3}}{2} \boxed{<} \sqrt{2}$

$1+\sqrt{3} \boxed{<} 2\sqrt{2}$

$1+2\sqrt{3}+3 \boxed{<} 8$

$2\sqrt{3} \boxed{<} 4$

$\sqrt{3} \boxed{<} 2$

Find & classify critical no.'s; Intervals of concavity / monotonicity; inflection pts.

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(12) $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x-2)(x+1)$

Critical numbers: $0, 2, -1.$ $\rightarrow$ Classify:

x		-1		0		2	
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$-$	0	$+$	$+$
$x+1$	$-$	$-$	0	$+$	$+$	$+$	$+$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

f: $\searrow$ min $\nearrow$ max $\searrow$ min $\nearrow$

at $x = -1$: local min

at $x = 0$: local max

at $x = 2$: local min

$f \nearrow$ on $(-1, 0) \cup (2, \infty)$

$f \searrow$ on $(-\infty, -1) \cup (0, 2)$

$f''(x) = 36x^2 - 24x - 24 = 12(3x^2 - 2x - 2)$

$\Delta = 4 + 8 \cdot 3 = 28 \Rightarrow x = \frac{2 \pm \sqrt{28}}{6}$

$\Rightarrow$

x		$\frac{2-\sqrt{28}}{6}$			$\frac{2+\sqrt{28}}{6}$	
$f''(x)$	$+$	0	$-$	$-$	0	$+$
$f(x)$	$\cup$		$\cap$			$\cup$

Inflection pts.: $x = \frac{2 \pm \sqrt{28}}{6}$

$f \cup$ on $\left(-\infty, \frac{2-\sqrt{28}}{6}\right) \cup \left(\frac{2+\sqrt{28}}{6}, \infty\right)$

$f \cap$ on $\left(\frac{2-\sqrt{28}}{6}, \frac{2+\sqrt{28}}{6}\right)$

(13) $f(x) = x^3 - 12x + 1$

$f'(x) = 3x^2 - 12 = 3(x^2 - 4) \Rightarrow$ Critical numbers: $-2, 2$

x		-2		2	
$f'(x)$	$+$	0	$-$	0	$+$
f	$\nearrow$	max	$\searrow$	min	$\nearrow$

$x = -2 \Rightarrow$ local max
$x = 2 \Rightarrow$ local min

$f \nearrow$ on $(-\infty, -2) \cup (2, \infty)$
$f \searrow$ on $(-2, 2)$

$f''(x) = 6x$

x			0		
f''	$-$	$-$	0	$+$	$+$
f	$\cap$			$\cup$	

$f \cup$ on $(0, \infty)$
$f \cap$ on $(-\infty, 0)$

$x = 0$ inflection pt.

(14) $f(x) = x(x - 8\sqrt{x}) = x^2 - 8x\sqrt{x} = x^2 - 8 \cdot x^{3/2}$

$f'(x) = 2x - \overset{4}{\cancel{8}} \cdot \frac{3}{\cancel{2}} x^{1/2} = 2x - 12\sqrt{x} = 2\sqrt{x}(\sqrt{x} - 6)$

root@ $x=0$ root@ $\sqrt{x}=6$; $x=36$

Critical numbers: $0, 36$

x	0		36		∞
$f'(x)$	0	$-$	0	$+$	$+$
$f(x)$		$\searrow$	min	$\nearrow$	

$x = 0$ not a local min/max
$x = 36$ local min

$f \nearrow$ on $(36, \infty)$
$f \searrow$ on $(0, 36)$

$f''(x) = 2 - 12 \cdot \frac{1}{2\sqrt{x}} = 2 - \frac{6}{\sqrt{x}} = \frac{2\sqrt{x} - 6}{\sqrt{x}}$ $\rightarrow$ always $\oplus$

$2\sqrt{x} - 6 = 0 \Leftrightarrow \sqrt{x} = 3 \Leftrightarrow x = 9$

x	0			9		
$f''(x)$	$\vert$	$-$	$-$	0	$+$	$+$
f		$\cap$			$\cup$	

$f \cup$ on $(9, \infty)$
$f \cap$ on $(0, 9)$
inflection pt. at $x = 9$

(15) $f(x) = x^2 - 3x$, which must be true by MVT?

Apply MVT to f on (0, 4).

$$\frac{f(4)-f(0)}{4-0} = \frac{(16-12)-0}{4} = 1 \Rightarrow \text{by MVT, there is } c \text{ in } (0,4) \text{ s.t. } f'(c)=1$$

(B)

(16) Find a positive value c for x that satisfies the conclusion of the MVT for $f(x) = 3x^2 - 5x + 1$ on the interval [2, 5].

$$f'(c) = \frac{f(5)-f(2)}{5-2} = \frac{(3\cdot 25 - 25 + 1) - (3\cdot 4 - 10 + 1)}{3} = \frac{51-3}{3} = \frac{48}{3} = \underline{\underline{16}}$$

$f'(x) = 6x - 5 \Rightarrow$ solve: $6c - 5 = 16$; $6c = 21$; $c = \frac{21}{6} = \frac{7}{2}$.

(17) $f(x) = \frac{1}{x} - 2$; MVT on [1, 3]

There is c in (1, 3) such that $f'(c) = \frac{f(3)-f(1)}{3-1} = \frac{(\frac{1}{3}-2)-(1-2)}{2}$

$$= \frac{\frac{1}{3} - \cancel{2} - 1 + \cancel{2}}{2} = \frac{-\frac{2}{3}}{2} = -\frac{1}{3}$$

$\Rightarrow$ The graph of f has a tangent line b/w 1 & 3 w/ slope $-\frac{1}{3}$ (B)

(18) Apply Newton's Method to approximate the root of $x^3 - 2x^2 - 1 = 0$. If we start at $x_1 = 2$, then after one iteration x_2 is?

$f(x) = x^3 - 2x^2 - 1$; $f(2) = 8 - 8 - 1 = -1$

$f'(x) = 3x^2 - 4x$; $f'(2) = 12 - 8 = 4$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2 - \frac{-1}{4} = 2 + \frac{1}{4} = \frac{9}{4}$$ (C