Exam 1 Prep - Worksheet 1. - Solutions

① Avg. ROC of $f(t) = 5 + \cos(t)$ over $t \in [0, \pi/4]$

Avg. ROC of $f(x)$ over $x \in [x_1, x_2]$: $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$\frac{f(\pi/4) - f(0)}{\pi/4 - 0} = \frac{\left(5 + \frac{\sqrt{2}}{2}\right) - (5+1)}{\pi/4} = \frac{\frac{\sqrt{2}}{2} - 1}{\frac{\pi}{4}} = \boxed{\frac{2\sqrt{2} - 4}{\pi}}$$

(2a) $$f(x) = \frac{3x - 12}{x^2 - 7x + 6} = \frac{3(x-4)}{(x-1)(x-6)}$$

Vertical Asymptotes: $x = 1, 6$
Domain of continuity: $x \in (-\infty, 1) \cup (1, 6) \cup (6, \infty)$.

(2b) $f(x) = \dfrac{x}{x^2+1}$ No vertical asymptotes (x^2+1 is never 0)
Domain of continuity: $(-\infty, \infty)$.

(2c) $$f(x) = \frac{3x - 6}{x^2 - 6x + 8} = \frac{3\cancel{(x-2)}}{\cancel{(x-2)}(x-4)} = \frac{3}{x-4} \text{ when } x \neq 2$$

Vertical Asymptote: $x = 4$
Domain of Continuity: $(-\infty, 2) \cup (2, 4) \cup (4, \infty)$.

! Always check for common factors b/w the numerator & denominator !

③ $$\lim_{x \to 2} \left(\frac{1}{x-2} + \frac{1}{|x-2|} \right)$$

$$|☺| = \begin{cases} -☺, & \text{when } ☺ < 0 \\ ☺, & \text{when } ☺ \geq 0 \end{cases}$$

! For any absolute value limits you must do side limits !

$$\lim_{x \to 2^-} \left(\frac{1}{x-2} + \frac{1}{|x-2|} \right) = \lim_{x \to 2^-} \underbrace{\left(\frac{1}{x-2} - \frac{1}{x-2} \right)}_{0} = \boxed{0}$$

$$|x-2| = \begin{cases} -(x-2), & \text{when } x - 2 < 0 \quad x < 2 \\ +(x-2), & \text{when } x - 2 \geq 0 \quad x \geq 2 \end{cases}$$

$$\lim_{x \to 2^+} \left(\frac{1}{x-2} + \frac{1}{|x-2|} \right) = \lim_{x \to 2^+} \left(\frac{1}{x-2} + \frac{1}{x-2} \right) = \lim_{x \to 2^+} \left(\frac{2}{x-2} \right) = \boxed{+\infty} \quad \left(\frac{2}{0_+} \right)$$

$\Rightarrow \lim_{x \to 2} \left(\frac{1}{x-2} + \frac{1}{|x-2|} \right)$ DNE b/c the side limits do not agree.

④ $$\lim_{x \to 2} (x-2) \cos\left(\frac{1}{x-2} \right)$$ Squeeze Theorem!

$-1 \leq \cos\left(\frac{1}{x-2}\right) \leq 1$ for all x

$\Rightarrow -(x-2) \leq (x-2)\cos\left(\frac{1}{x-2}\right) \leq (x-2)$ for all x

$$-(x-2) \xrightarrow{x \to 2} 0 \xleftarrow{x \to 2} (x-2)$$

$\Rightarrow \lim_{x \to 2} (x-2)\cos\left(\frac{1}{x-2}\right) = \boxed{0}$ by the Squeeze Thm.

(5) $\ell(t)$ = length @ time t
$w(t)$ = width @ time t
$A(t)$ = area @ time t

$\ell'(t) = 8$
$w'(t) = 3$
$\left.\frac{dA}{dt}\right|_{\ell=20,\, w=10} = ?$

$$A(t) = \ell(t)\, w(t)$$

$$A'(t) = \underbrace{\ell'(t)}_{8}\, \underbrace{w(t)}_{10} + \underbrace{\ell(t)}_{20}\, \underbrace{w'(t)}_{3} = 80+60 = \boxed{140}\ cm^2/s.$$

(6) $f(x) = \begin{cases} x^2-8, & \text{if } x \leq c \\ 10x-33, & \text{if } x > c. \end{cases}$

$\left.\begin{array}{l} \lim_{x\to c-} f(x) = c^2-8 \\ \lim_{x\to c+} f(x) = 10c-33 \\ f(c) = c^2-8 \end{array}\right\}$ must be equal for f to be continuous @ $x=c$.

$$c^2-8 = 10c-33$$
$$c^2-10c+25 = 0$$
$$(c-5)^2 = 0 \quad \Rightarrow \quad \boxed{c=5}$$

(7) $$M = \frac{a^2\sqrt{b}\, p^{-1/4}}{z^7}$$

$$\frac{dM}{da} = \frac{2a\sqrt{b}\, p^{-1/4}}{z^7}$$

$$\frac{dM}{db} = \frac{a^2 p^{-1/4}}{z^7} \cdot \frac{1}{2\sqrt{b}}$$

$$\frac{dM}{dp} = \frac{a^2\sqrt{b}}{z^7}\left(-\frac{1}{4}\right)p^{-5/4}$$

$$\frac{dM}{dz} = \left(a^2\sqrt{b}\, p^{-1/4}\right)(-7)z^{-8}.$$