1.4 **Average ROC** of a function f over an interval $[a,b]$: $\boxed{\dfrac{f(b)-f(a)}{b-a}}$

1.5 Limit of a function:

* determine (side) limits from a graph (WW 1,2,3)
* determine (side) limits of branch functions (WW 4,5)
* Vertical Asymptotes (Remember to also factor the numerator & check common factors
* Infinite limits at vertical asymptote points $\left(\frac{1}{0}\right)$ – see below

1.6 Limit Laws:

$\boxed{\frac{0}{0} \text{ limits}}$ — **cancel common factors:**

- $\displaystyle\lim_{x\to 2} \frac{x^2+5x-14}{x^2-2x} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+7)}{x\cancel{(x-2)}} = \frac{9}{2}.$
- $\displaystyle\lim_{x\to 64} \frac{\sqrt{x}-8}{x-64} = \lim_{x\to 64} \frac{\cancel{(\sqrt{x}-8)}}{\cancel{(\sqrt{x}-8)}(\sqrt{x}+8)} = \frac{1}{16}$

$a^2-b^2=(a-b)(a+b)$

$x-y=(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})$

use: $\displaystyle\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1.$

- $\displaystyle\lim_{x\to 4} \frac{\sin(x^2-16)}{x-4} = \lim_{x\to 4} \frac{\sin(x^2-16)}{(x-4)}\cdot\frac{(x+4)}{(x+4)}$

$\displaystyle = \lim_{x\to 4}\left(\frac{\sin(x^2-16)}{(x^2-16)}\cdot(x+4)\right) = 1\cdot 8 = \boxed{8}$

$\boxed{\frac{1}{0} \text{ limits}}$ — **MUST do side limits and determine the sign of $\pm\infty$**

- $\displaystyle\lim_{x\to 2}\left(\frac{x-3}{x-2}\right)$: $\left(\frac{-1}{0}\right)$: $\displaystyle\lim_{x\to 2^-}\left(\frac{x-3}{x-2}\right) = \boxed{+\infty}$ $\left(\frac{-1}{0^-}\right)$; $\displaystyle\lim_{x\to 2^+}\left(\frac{x-3}{x-2}\right) = \boxed{-\infty}$ $\left(\frac{-1}{0^+}\right)$

$\boxed{\textbf{Absolute Value limits}}$ — **MUST do side limits and use appropriate definition of $|x|$:**

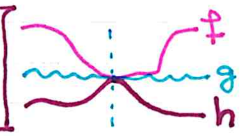
$$|f(x)| = \begin{cases} -f(x), & \text{if } f(x)<0 \\ +f(x), & \text{if } f(x)\geq 0 \end{cases}$$

- $\displaystyle\lim_{x\to 6} \frac{x(x-6)}{|x-6|}$: $|x-6| = \begin{cases} -(x-6), & \text{if } (x-6)<0, \text{ or } x<6 \\ +(x-6), & \text{if } (x-6)\geq 0, \text{ or } x\geq 6 \end{cases}$

$\displaystyle\lim_{x\to 6^-} \frac{x(x-6)}{|x-6|} = \lim_{x\to 6^-} \frac{x\cancel{(x-6)}}{-\cancel{(x-6)}} = -6$; $\displaystyle\lim_{x\to 6^+} \frac{x(x-6)}{|x-6|} = \lim_{x\to 6^+} \frac{x\cancel{(x-6)}}{\cancel{(x-6)}} = 6.$

***Squeeze Theorem:**

> If $f(x)\leq g(x)\leq h(x)$ when x is near a, and $\displaystyle\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\displaystyle\lim_{x\to a} g(x) = L.$



- $\displaystyle\lim_{x\to 9} (x-9)\sin\left(\frac{1}{x-9}\right) = 0$: by the Squeeze Theorem.

$-1 \leq \sin\left(\frac{1}{x-9}\right) \leq 1$

$-(x-9) \leq \pm(x-9)\sin\left(\frac{1}{x-9}\right) \leq (x-9)$

$\xrightarrow{x\to 9} 0 \xleftarrow{x\to 9}$

1.8 Continuity:

* Definition: f is continuous at $x=a$ if $\lim_{x\to a} f(x) = f(a)$.
* Determine intervals of continuity;
* Types of discontinuities:
  * **Jump**: $\lim_{x\to a-} f(x) \neq \lim_{x\to a+} f(x)$
  * **Removable**: $\lim_{x\to a} f(x) \neq f(a)$
  * **Infinite**: At least one of $\lim_{x\to a\pm} f(x)$ is $\pm\infty$.

* **Intermediate Value Thm.:**

> If f is continuous on $[a,b]$, and N is any number b/w $f(a)$ & $f(b)$, then there is a value $c \in (a,b)$ such that $f(c)=N$.

2.1 2.2 Derivatives:

* Find the derivative of f using the definition:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

* Eqn. of tangent line to the graph of f at $(x_0, f(x_0))$:

$$y - f(x_0) = f'(x_0)(x - x_0).$$

* Higher derivatives; Leibniz notation.
* DIFFERENTIABLE $\Rightarrow$ CONTINUOUS but CONTINUOUS $\not\Rightarrow$ DIFFERENTIABLE

2.3 Differentiation Formulas

* $\boxed{\frac{d}{dx}(c)=0}$; $\boxed{\frac{d}{dx}(x^n)=nx^{n-1}}$
* Algebraic Rules: $\boxed{\frac{d}{dx}(c\cdot f) = c\cdot \frac{df}{dx}}$; $\boxed{\frac{d}{dx}[f\pm g] = \frac{df}{dx} \pm \frac{dg}{dx}}$
* Product Rule: $\boxed{(fg)' = f'g + fg'}$ * Quotient Rule: $\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$

2.4 Derivatives of Trig. Functions: (Know the formulas)

2.5 Chain Rule (Practice many problems) $(f\circ g)' = f'(g)\cdot g'$

2.6 Implicit Diff. (Practice many problems) $\frac{d}{dx}(xy^9) = y^9 + 9xy^8 \frac{dy}{dx}$

2.7 Rates of Change

* $s(t)$ = pos.; $v(t) = s'(t)$ = velocity; $a(t) = v'(t) = s''(t)$ = acceleration
* Movement in $\oplus/\ominus$ direction: sign of $v(t)$
* Max/min height when $v(t)=0$
* Careful w/ speed vs. velocity.

2.8 Related Rates:

1. Draw a picture;
2. Name your variables;
3. Find the equation that connects the variables;
4. Differentiate both sides of the eqn. in 3 wrt. time
5. Plug in known values in 4.