Chain Rule: Extra Practice Problems

Find the derivatives of the functions below:

1. $y = \sqrt{x^3 + 1}$

2. $y = \cos(x^2)$

3. $y = \tan\left(\dfrac{x}{x+1}\right)$

4. $y = (t^2 + 3t + 1)^{-5/2}$

5. $y = \cos^7\left(x^{-1/4}\right)$

6. $y = \dfrac{1}{\sqrt{\cos(x^2) + 1}}$

7. $y = x\cos(1 - 3x)$

8. $y = (x^3 + \cos(x))^{-4}$

9. $y = \sqrt{\sin x \cos x}$

10. $y = \big(\cos(6x) + \sin(x^2)\big)^{1/2}$

11. $y = \tan^3 x + \tan(x^3)$

12. $y = \sec^4(3x)$

① $y=\sqrt{x^3+1}$

$$\sqrt{f(x)} \mapsto \frac{1}{2\sqrt{f(x)}}\cdot f'(x)$$

$$y' = \underbrace{\frac{1}{2\sqrt{x^3+1}}}_{\text{derivative of outside function } \sqrt{\ }} \cdot \underbrace{(3x^2)}_{\text{derivative of inside function } (x^3+1).}$$

② $y=\cos(x^2)$

$$\cos(f(x)) \mapsto -\sin(f(x))\cdot f'(x)$$

$$y' = \underbrace{-\sin(x^2)}_{\text{derivative of outside function } \underline{\cos}} \cdot \underbrace{(2x)}_{\text{derivative of inside function } (x^2).}$$

③ $y=\tan\left(\frac{x}{x+1}\right)$

$$\tan(f(x)) \mapsto \sec^2(f(x))\cdot f'(x)$$

$$y' = \underbrace{\sec^2\left(\frac{x}{x+1}\right)}_{\text{derivative of outside function } \underline{\tan}} \cdot \underbrace{\frac{1\cdot(x+1)-x\cdot(1)}{(x+1)^2}}_{\text{derivative of inside function } \frac{x}{x+1}} = \sec^2\left(\frac{x}{x+1}\right)\cdot\frac{1}{(x+1)^2}$$

④ $y=(t^2+3t+1)^{-5/2}$

$$(f(x))^a \mapsto a\,(f(x))^{a-1}\cdot f'(x)$$

$$y' = \underbrace{-\frac{5}{2}(t^2+3t+1)^{-7/2}}_{\text{derivative of outside power function } f^{-5/2}} \cdot \underbrace{(2t+3)}_{\text{derivative of inside function } (t^2+3t+1).}$$

⑤ $y=\cos^7(x^{-1/4})$

$$=\left(\cos(x^{-1/4})\right)^7$$

$$\cos^a(f(x)) = [\cos(f(x))]^a \mapsto a\cos^{a-1}(f(x))\cdot(-\sin(f(x)))\cdot f'(x$$

$$y' = \underbrace{7\cos^6(x^{-1/4})}_{\text{derivative of \underline{power} function } (\ )^7} \cdot \underbrace{-\sin(x^{-1/4})}_{\text{derivative of \underline{cos} function}} \cdot \underbrace{-\frac{1}{4}x^{-5/4}}_{\text{derivative of inside function } (x^{-1/4})}$$

$$\underbrace{7^{th}\text{ power}}\left(\underbrace{\cos}\left(\underbrace{x^{-1/4}}\right)\right)$$

⑥ $y = \dfrac{1}{\sqrt{\cos(x^2)+1}} = \left(\cos(x^2)+1\right)^{-1/2}$

$y' = -\frac{1}{2}\left(\cos(x^2)+1\right)^{-3/2} \cdot \left(-\sin(x^2)\cdot 2x\right).$

⑦ $y = x\cos(1-3x)$ (Product Rule)

$y' = (x)'\cdot\cos(1-3x) + x\cdot\left(\cos(1-3x)\right)'$
$= \cos(1-3x) + x\cdot\left(-\sin(1-3x)\cdot(-3)\right)$
$= \cos(1-3x) + 3x\sin(1-3x).$

⑧. $y = (x^3+\cos x)^{-4}$

$y' = -4(x^3+\cos x)^{-5}\cdot(3x^2-\sin x).$

⑨ $y = \sqrt{\sin x\cdot\cos x}$

$y' = \dfrac{1}{2\sqrt{\sin x\cos x}}\left(\cos x\cdot\cos x + \sin x\cdot(-\sin x)\right) = \dfrac{\cos^2 x - \sin^2 x}{2\sqrt{\sin x\cos x}}$

⑩ $y = \left(\cos(6x)+\sin(x^2)\right)^{1/2}$

$y' = \frac{1}{2}\left(\cos(6x)+\sin(x^2)\right)^{-1/2}\cdot\left(-\sin(6x)\cdot 6 + \cos(x^2)\cdot 2x\right)$

⑪ $y = \tan^3 x + \tan(x^3)$

$y' = \underbrace{3\tan^2 x\cdot\sec^2 x}_{(\tan^3 x)'} + \underbrace{\sec^2(x^3)\cdot 3x^2}_{(\tan(x^3))'}$

⑫ $y = \sec^4(3x)$

$y' = \underbrace{4\sec^3(3x)}_{\text{from }(\ )^4}\cdot\underbrace{\sec(3x)\tan(3x)}_{\text{from sec}}\cdot\underbrace{3}_{\text{from }3x}$

$\underbrace{4^{\text{th}}\text{ power}}\left(\underbrace{\sec}\ \underbrace{(3x)}\right)$