EVALUATE THE SUMS

1.) $$\sum_{n=0}^{\infty} \frac{2+3^n}{5^n} = 2\sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n + \sum_{n=0}^{\infty}\left(\frac{3}{5}\right)^n$$

$$= 2\cdot \frac{1}{1-\frac{1}{5}} + \frac{1}{1-\frac{3}{5}}$$

$$= 2\cdot \frac{5}{4} + \frac{5}{2} = \boxed{5}$$

2.) $$\sum_{n=0}^{\infty} \frac{1+3^{n-1}}{2^{2n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2^{2n+1}} + \sum_{n=0}^{\infty} \frac{3^{n-1}}{2^{2n+1}}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty} \frac{1}{2^{2n}} + \frac{1}{2}\,\frac{1}{3}\sum_{n=0}^{\infty} \frac{3^n}{2^{2n}}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n + \frac{1}{6}\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n$$

$$= \frac{1}{2}\cdot \frac{1}{1-\frac{1}{4}} + \frac{1}{6}\,\frac{1}{1-\frac{3}{4}}$$

$$= \frac{1}{2}\cdot\frac{4}{3} + \frac{1}{6}\cdot 4$$

$$= \frac{2}{3} + \frac{2}{3} = \boxed{\frac{4}{3}}$$

3.) $$\sum_{n=2}^{\infty} e^{3-2n} = e^3 \sum_{n=2}^{\infty} \frac{1}{e^{2n}}$$

$$= e^3\left(\frac{1}{e^4} + \frac{1}{e^6} + \frac{1}{e^8} + \dots\right)$$

$$= e^3\,\frac{1}{e^4}\left(1 + \frac{1}{e^2} + \frac{1}{e^4} + \dots\right)$$

$$= \frac{1}{e}\sum_{n=0}^{\infty}\left(\frac{1}{e^2}\right)^n$$

$$= \frac{1}{e}\,\frac{1}{1-\frac{1}{e^2}} = \frac{1}{e}\,\frac{e^2}{e^2-1} = \boxed{\frac{e}{e^2-1}}$$

4.) $$\frac{64}{49} + \frac{8}{7} + \underbrace{1 + \frac{7}{8} + \frac{49}{64} + \frac{343}{512} + \dots}_{\sum_{n=0}^{\infty}\left(\frac{7}{8}\right)^n}$$

$$= \frac{64}{49} + \frac{8}{7} + \sum_{n=0}^{\infty}\left(\frac{7}{8}\right)^n$$

$$= \frac{120}{49} + \frac{1}{1-\frac{7}{8}} = \frac{120}{49} + 8 = \boxed{\frac{512}{49}}$$