FIND THE LIMITS OF THE SEQUENCES:

1. $a_n = \cos(\pi n)$

$\lim_{n\to\infty} a_n$ DNE

2. $a_n = 3 + \left(-\frac{1}{2}\right)^n$

$\lim_{n\to\infty} a_n = \boxed{3}$

$\lim_{n\to\infty} \left(-\frac{1}{2}\right)^n = 0$ b/c $|r| = \left|-\frac{1}{2}\right| < 1$

3. $a_n = \dfrac{e^n + 3^n}{5^n}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\left(\frac{e}{5}\right)^n + \left(\frac{3}{5}\right)^n\right) = \boxed{0}$

b/c $\left|\frac{e}{5}\right| < 1$ and $\left|\frac{3}{5}\right| < 1$.

4. $a_n = -2^{-n}$

$\lim_{n\to\infty} a_n = -\lim_{n\to\infty} \frac{1}{2^n} = \boxed{0}$

5. $\lim_{n\to\infty} \dfrac{(-1)^n n^2 + n}{4n^2 + 1}$ DNE

(oscillates b/w +4 and −4).

6. $\lim_{n\to\infty} \dfrac{3n^{5/2} + 4}{n\sqrt{2n^3 + 1}} = \boxed{\dfrac{3}{\sqrt{2}}}$

(leading power is 5/2 on both)

7. $\lim_{n\to\infty} \dfrac{n^3 + 2e^{-n}}{3n^3 + 4e^{-n}}$ $\cdot \dfrac{e^n}{e^n}$

$= \lim_{n\to\infty} \dfrac{n^3 e^n + 2}{3n^3 e^n + 4}$

$\lim_{x\to\infty} \dfrac{x^3 e^x + 2}{3x^3 e^x + 4} \overset{\frac{\infty}{\infty}}{\overset{\text{l'H}}{=}} \lim_{x\to\infty} \dfrac{3x^2 e^x + x^3 e^x}{9x^2 e^x + 3x^3 e^x}$

$= \lim_{x\to\infty} \dfrac{3x^2 + x^3}{9x^2 + 3x^3} = \boxed{\dfrac{1}{3}}$