FIND THE INTERVAL OF CONVERGENCE

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x-5)^n = \sum_{n=1}^{\infty} a_n$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{n+1}\cdot\frac{n}{(x-5)^n}\right|$$

$$= \lim_{n\to\infty}\frac{|x-5|n}{n+1} = |x-5| < 1$$

=> By the Ratio Test, $\boxed{R=1}$ and the series converges for

$-1 < x-5 < 1$; $4 < x < 6$

Endpoints:

$\underline{x=4}$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}(-1)^n = \sum_{n=1}^{\infty}\frac{1}{n}$

divergent (harmonic series)

$\underline{x=6}$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cdot 1^n = \sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

convergent (alternating harmonic)

=> $\boxed{\begin{array}{l} R=1 \\ I=(4,6] \end{array}}$

2. $\sum_{n=1}^{\infty}\frac{(x-4)^n}{n^4}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-4)^{n+1}}{(n+1)^4}\;\frac{n^4}{(x-4)^n}\right|$$

$$= \lim_{n\to\infty}\frac{n^4}{(n+1)^4}|x-4|$$

$$= |x-4| < 1$$

=> By the Ratio Test, $\boxed{R=1}$ and the series converges for

$-1 < x-4 < 1$; $3 < x < 5$.

Endpoints:

$\underline{x=3}$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}$

$\underline{x=5}$: $\sum_{n=1}^{\infty}\frac{1}{n^4}$

Both are absolutely convergent (=> convergent) because $\sum_{n=1}^{\infty}|a_n| = \sum_{n=1}^{\infty}\frac{1}{n^4}$ convergent p-series ($p=4>1$)

=> $\boxed{\begin{array}{l} R=1 \\ I=[3,5] \end{array}}$