$$\boxed{\begin{aligned} \sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad \forall x \\ \cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \quad \forall x \\ e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \forall x \end{aligned}}$$

→ Use these to find the Taylor series below:

1. $f(x) = x^2 e^x$, Maclaurin series

$$x^2 e^x = x^2 \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n+2}}{n!}$$

$$= \sum_{n=2}^{\infty} \frac{x^n}{(n-2)!}$$

$$x^2 + \frac{x^3}{1!} + \frac{x^4}{2!} + \frac{x^5}{3!} + \ldots$$

2. $f(x) = e^{-x^2}$, Maclaurin series.

Replace x by $(-x^2)$ in the Maclaurin series of e^x:

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$$

3). $A = \int_0^1 \sin(x^2)\,dx$ Express as an infinite series!

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\Rightarrow \sin(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+2}$$

$$\Rightarrow A = \int_0^1 \sin(x^2)\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int_0^1 x^{4n+2}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left.\frac{x^{4n+3}}{4n+3}\right|_0^1$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{1}{4n+3}\right)}$$