Consider the function $\boxed{f(x)=\frac{1}{x}}$

1). Find a power series representation of the form

$$f(x)=\sum_{n=0}^{\infty} c_n (x-2)^n$$

by using the geometric series. (+ Interval of conv.)

Hint: Write f as: $f(x)=\frac{1}{x}=\frac{1}{2+(x-2)}$

$$f(x)=\frac{1}{x}=\frac{1}{2+(x-2)}=\frac{1}{2}\,\frac{1}{1+\frac{x-2}{2}}$$

$$=\frac{1}{2}\,\frac{1}{1-\left(-\frac{x-2}{2}\right)}$$

$$=\frac{1}{2}\sum_{n=0}^{\infty}\left(-\frac{x-2}{2}\right)^n, \quad \forall \left|\frac{x-2}{2}\right|<1$$

$$|x-2|<2 \quad (R=2)$$

$$=\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}(x-2)^n \qquad -2<x-2<2$$

$$\boxed{0<x<4}$$

$$\boxed{\frac{1}{x}=\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}(x-2)^n, \quad \forall x\in(0,4)}$$

2). Now use Taylor series formula

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

to obtain the Taylor series of $f(x)=\frac{1}{x}$ about $x=2$.

$$f(x)=\frac{1}{x}=x^{-1}; \quad f(2)=\frac{1}{2} \quad =+\frac{0!}{2}$$

$$f'(x)=\frac{-1}{x^2}=-x^{-2}; \quad f'(2)=\frac{-1}{2^2} \quad =-\frac{1!}{2^2}$$

$$f''(x)=\frac{2}{x^3}=2x^{-3}; \quad f''(2)=\frac{+2}{2^3} \quad =+\frac{2!}{2^3}$$

$$f'''(x)=\frac{-2\cdot 3}{x^4}; \quad f'''(2)=-\frac{2\cdot 3}{2^4} \quad =-\frac{3!}{2^4}$$

$$f^{IV}(x)=\frac{+2\cdot 3\cdot 4}{x^5}; \quad f^{IV}(2)=+\frac{2\cdot 3\cdot 4}{2^5}=+\frac{4!}{2^5}$$

$$\boxed{f^{(n)}(2)=(-1)^n\cdot\frac{n!}{2^{n+1}}}$$

⇒ Taylor series about $x=2$:

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(2)}{n!}(x-2)^n=\sum_{n=0}^{\infty}\frac{1}{n!}(-1)^n\frac{n!}{2^{n+1}}(x-2)^n$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}(x-2)^n$$

← same answer!