FIND A POWER SERIES REPRESENTATION FOR THE FUNCTIONS BELOW (+ INTERVAL OF CONV.)

1). $f(x) = \dfrac{x}{x^2+16}$

$$f(x) = \frac{x}{16}\,\frac{1}{1+\frac{x^2}{16}} = \frac{x}{16}\,\frac{1}{1-\left(-\frac{x^2}{16}\right)}$$

$$= \frac{x}{16}\sum_{n=0}^{\infty}\left(-\frac{x^2}{16}\right)^n, \quad \left|\frac{x^2}{16}\right|<1$$

$$= \frac{x}{16}\sum_{n=0}^{\infty}\frac{(-1)^n}{16^n}x^{2n} \quad |x^2|<16$$

$$|x|<4$$

$$\Rightarrow \boxed{\frac{x}{x^2+16} = \sum_{n=0}^{\infty}\frac{(-1)^n}{16^{n+1}}x^{2n+1}, \ \forall x\in(-4,4)}$$

2). $f(x) = \ln(5-x)$

$$\left(\ln(5-x)\right)' = \frac{-1}{5-x}$$

$$f(x) = \ln(5-x) = -\int\frac{1}{5-x}\,dx$$

$$= -\frac{1}{5}\int\frac{1}{1-x/5}\,dx$$

$$= -\frac{1}{5}\int\left(\sum_{n=0}^{\infty}\left(\frac{x}{5}\right)^n\right)dx, \quad \left|\frac{x}{5}\right|<1;\ |x|<5$$

$$= -\frac{1}{5}\sum_{n=0}^{\infty}\frac{1}{5^n}\int x^n\,dx$$

$$= -\frac{1}{5}\sum_{n=0}^{\infty}\frac{1}{5^n}\frac{x^{n+1}}{n+1} + C$$

$$\ln(5-x) = \sum_{n=0}^{\infty}\left(-\frac{1}{5^{n+1}}\frac{x^{n+1}}{n+1}\right) + C, \quad \text{for all } x\in(-5,5)$$

$\Rightarrow$ Let $x=0$: $\ln(5) = C$

$$\Rightarrow \boxed{\begin{aligned}\ln(5-x) &= \ln(5) - \sum_{n=0}^{\infty}\frac{1}{5^{n+1}}\frac{x^{n+1}}{n+1} \\ &= \ln(5) - \sum_{n=1}^{\infty}\frac{1}{5^n}\frac{x^n}{n}\end{aligned} \quad \forall x\in(-5,5)}$$