FIND A POWER SERIES REPRESENTATION FOR THE FUNCTIONS BELOW (+INTERVAL OF CONV.)

1). $f(x) = \dfrac{x}{9+x^2}$

$$f(x) = \frac{x}{9} \cdot \frac{1}{1+\frac{x^2}{9}} = \frac{x}{9} \cdot \frac{1}{1-\left(-\frac{x^2}{9}\right)}$$

$$= \frac{x}{9} \cdot \sum_{n=0}^{\infty} \left(-\frac{x^2}{9}\right)^n, \quad \forall \left|\frac{x^2}{9}\right| < 1$$

$$= \frac{x}{9} \sum_{n=0}^{\infty} (-1)^n \frac{1}{9^n} x^{2n} \qquad |x^2| < 9$$

$$|x| < 3$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{9^{n+1}} x^{2n+1}$$

$$\boxed{\frac{x}{9+x^2} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{9^{n+1}} x^{2n+1}, \quad \forall |x| < 3}$$

$R=3$, $I=(-3,3)$.

2). $f(x) = \dfrac{1+x}{(1-x)^2}$

$$\left(\frac{1}{1-x}\right)' = \frac{1}{(1-x)^2}, \quad \forall |x| < 1$$

$$\Rightarrow \frac{1}{(1-x)^2} = \left(\sum_{n=0}^{\infty} x^n\right)'$$

$$= \sum_{n=1}^{\infty} n x^{n-1}, \quad \forall |x| < 1$$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} n x^n = x + 2x^2 + 3x^3 + \dots$$

$\oplus$ $$\frac{1+x}{(1-x)^2} = 1 + (2+1)x + (3+2)x^2 + (4+3)x^3 + \dots$$

$$= \sum_{n=0}^{\infty} ((n+1)+n) x^n$$

$$= \sum_{n=0}^{\infty} (2n+1) x^n$$

$$\boxed{\frac{1+x}{(1-x)^2} = \sum_{n=0}^{\infty} (2n+1) x^n, \quad \forall x \in (-1,1)}$$

$R=1$

$I=(-1,1)$.