FIND THE LIMIT OF THE SEQUENCES:

1). $a_n = \dfrac{4-2n}{2+4n}$

$\lim\limits_{n\to\infty} a_n = \dfrac{-2}{4} = \dfrac{-1}{2}$

2). $a_n = 9 + (-1)^n \cdot 4$

$\lim\limits_{n\to\infty} a_n$ DNE

3). $a_n = \dfrac{\ln\left(1-\frac{2}{n}\right)}{\sin\left(\frac{6}{n}\right)}$

Needs l'Hospital! $\left(\frac{0}{0}\right)$

$$\lim_{x\to\infty} \frac{\ln\left(1-\frac{2}{x}\right)}{\sin\left(\frac{6}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1-\frac{2}{x}} \cdot (-1) \cdot \cancel{\frac{-2}{x^2}}}{\cos\left(\frac{6}{x}\right) \cdot \cancel{\frac{-6}{x^2}}}$$

$$= \lim_{x\to\infty} - \frac{\frac{2}{1-2/x}}{6\cos\left(\frac{6}{x}\right)} = -\frac{2}{6} = -\frac{1}{3}$$

$\lim\limits_{n\to\infty} a_n = -\dfrac{1}{3}$

4). $a_n = \dfrac{4\cos(n^5)}{\sqrt{n}}$

Squeeze Theorem:

$$\frac{-4}{\sqrt{n}} \le \frac{4\cos(n^5)}{\sqrt{n}} \le \frac{4}{\sqrt{n}}$$

$\frac{-4}{\sqrt{n}} \xrightarrow{n\to\infty} 0 \xleftarrow{n\to\infty} \frac{4}{\sqrt{n}}$

$\boxed{\lim\limits_{n\to\infty} a_n = 0}$

5). $a_n = \left(\frac{1}{9}\right)^n + \dfrac{1}{\sqrt{5^n}}$

$= \left(\frac{1}{9}\right)^n + \left(\frac{1}{\sqrt{5}}\right)^n$

$\Rightarrow \boxed{\lim\limits_{n\to\infty} a_n = 0}$

Remember:

$$\lim_{n\to\infty} (r^n) = \begin{cases} 0, & \text{if } -1 < r < 1 \\ 1, & \text{if } r = 1 \\ \infty, & \text{if } r > 1 \\ \text{DNE}, & \text{if } r \le -1. \end{cases}$$

6). $a_n = \ln(n^2+1) - \ln(3n^2-n+1)$

$= \ln\left(\dfrac{n^2+1}{3n^2-n+1}\right)$

$\Rightarrow \lim\limits_{n\to\infty} a_n = \ln\left(\frac{1}{3}\right)$ b/c $\lim\limits_{n\to\infty} \dfrac{n^2+1}{3n^2-n+1} = \dfrac{1}{3}$

7). $a_n = \ln(n^2+1) - \ln(n+1)$

$= \ln\left(\dfrac{n^2+1}{n+1}\right)$

$\Rightarrow \lim\limits_{n\to\infty} a_n = \infty$ b/c $\lim\limits_{n\to\infty} \dfrac{n^2+1}{n+1} = \infty$

8). $a_n = \ln(n^2+n+1) - \frac{1}{2}\ln(3n^4+n^3)$

$= \ln(n^2+n+1) - \ln\sqrt{3n^4+n^3}$

$= \ln \dfrac{n^2+n+1}{\sqrt{3n^4+n^3}}$

$\Rightarrow \boxed{\lim\limits_{n\to\infty} a_n = \ln\left(\frac{1}{\sqrt{3}}\right)},$

b/c $\lim\limits_{n\to\infty} \dfrac{n^2+n+1}{\sqrt{3n^4+n^3}} = \dfrac{1}{\sqrt{3}}.$