FIND RADIUS & INTERVAL OF CONVERGENCE

1). $\sum_{n=1}^{\infty} \frac{n}{8^n}(x+3)^n$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{n+1}{8^{n+1}}(x+3)^{n+1}\cdot\frac{8^n}{n(x+3)^n}\right|$$

$$= \lim_{n\to\infty} |x+3|\,\frac{1}{8}\,\frac{n+1}{n} = \frac{|x+3|}{8} < 1$$

$\Rightarrow$ By Ratio Test, series is a.c. when $|x+3|<8$ and divergent when $|x+3|>8$. $R=8$

$|x+3|<8 \iff -8<x+3<8 \iff -11<x<5$

Endpoints:

$x=-11$: $\sum_{n=1}^{\infty}\frac{n}{8^n}(-8)^n = \sum_{n=1}^{\infty}(-1)^n n$ divergent

by T for Div: $\lim_{n\to\infty}\left((-1)^n n\right)$ DNE

$x=5$: $\sum_{n=1}^{\infty}\frac{n}{8^n}\cdot 8^n = \sum_{n=1}^{\infty} n$ divergent

by T for Div: $\lim_{n\to\infty}(n)=\infty$.

$$\Rightarrow \boxed{\begin{array}{l} R=8 \\ I=(-11,5)\end{array}}$$

2). $\sum_{n=1}^{\infty} n!\,(2x-1)^n$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!\,(2x-1)^{n+1}}{n!\,(2x-1)^n}\right|$$

$$= \lim_{n\to\infty}(n+1)\cdot|2x-1| = \begin{cases}\infty & \text{if } x\neq 1/2 \\ 0 & \text{if } x=1/2\end{cases}$$

$\Rightarrow$ By Ratio Test, series only converges if $x=1/2$

$$\boxed{R=0,\quad I=\{1/2\}}$$

3). $\sum_{n=1}^{\infty}\frac{(x-4)^n}{n^n}$ Root Test!

$$\lim_{n\to\infty}\sqrt[n]{|a_n|} = \lim_{n\to\infty}\frac{|x-4|}{n} = 0 < 1 \text{ for all real } x$$

$\Rightarrow$ By Root Test, the series is abs. conv. for all real x

$$\Rightarrow \boxed{\begin{array}{l} R=\infty \\ I=(-\infty,\infty)=\mathbb{R}\end{array}}$$

4). $\sum_{n=1}^{\infty}\frac{(-4)^n}{n\sqrt{n}}\cdot x^n$

$$a_n = \frac{(-4)^n}{n\sqrt{n}}\cdot x^n$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-4)^{n+1}\cdot x^{n+1}}{(n+1)\sqrt{n+1}}\cdot\frac{n\sqrt{n}}{(-4)^n\cdot x^n}\right|$$

$$= \lim_{n\to\infty} 4|x|\,\frac{n\sqrt{n}}{(n+1)\sqrt{n+1}} = 4|x| < 1$$

$\Rightarrow$ By Ratio Test, the series is abs. conv. for $|x|<\frac{1}{4}$ and divergent for $|x|>\frac{1}{4}$. $R=1/4$

Endpoints:

$x=-\frac{1}{4}$ $\Rightarrow$ $\sum_{n=1}^{\infty}\frac{(-4)^n}{n\sqrt{n}}\left(\frac{-1}{4}\right)^n = \sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$ convergent p-series ($p=3/2$)

$x=1/4$ $\Rightarrow$ $\sum_{n=1}^{\infty}\frac{(-4)^n}{n\sqrt{n}}\cdot\left(\frac{1}{4}\right)^n = \sum_{n=1}^{\infty}\frac{(-1)^n}{n\sqrt{n}}$ convergent

$\to$ AST: $\sum_{n=1}^{\infty}(-1)^n b_n$, w/ $b_n=\frac{1}{n\sqrt{n}}$ (dec, $\lim_{n\to\infty} b_n=0$)

or $\to$ Abs. conv.: $\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{n\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$ conv. p-series

$$\Rightarrow \boxed{\begin{array}{l} R=1/4 \\ I=[-1/4,1/4]\end{array}}$$