FIND RADIUS & INTERVAL OF CONVERGENCE

1). $\sum_{n=1}^{\infty} \frac{x^n}{9n-1}$

$a_n = \frac{x^n}{9n-1}$; $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{9n+8} \cdot \frac{9n-1}{x^n}\right|$

$= \lim_{n\to\infty} |x| \left(\frac{9n-1}{9n+8}\right) = |x| < 1$

=> By Ratio Test, series converges absolutely for $|x|<1$ and diverges for $|x|>1$.

Endpoints: $x=-1$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{9n-1}$ converges by AST

$= \sum_{n=1}^{\infty} (-1)^n b_n$, where $b_n = \frac{1}{9n-1}$

* $b_{n+1} = \frac{1}{9n+8} \leq \frac{1}{9n-1} = b_n$
* $\lim_{n\to\infty} b_n = 0$.

$x=1$: $\sum_{n=1}^{\infty} \frac{1}{9n-1}$ diverges by Comparison Test

$\frac{1}{9n-1} > \frac{1}{9n}$ => $\sum_{n=1}^{\infty} \frac{1}{9n-1} \geq \sum_{n=1}^{\infty} \frac{1}{9n} = \infty$ (harmonic series)

=> Radius of convergence: $R=1$
Interval of convergence: $I = [-1, 1)$

2). $\sum_{n=1}^{\infty} \frac{6^n x^n}{n^4}$

$a_n = \frac{6^n \cdot x^n}{n^4}$;

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{6^{n+1}x^{n+1}}{(n+1)^4} \cdot \frac{n^4}{6^n \cdot x^n}\right|$

$= \lim_{n\to\infty} |x| \underbrace{\frac{6n^4}{(n+1)^4}}_{\downarrow n\to\infty \; 6} = 6|x| < 1$

=> By Ratio Test, the series is absolutely convergent for $|x| < \frac{1}{6}$, and divergent for $|x| > \frac{1}{6}$.

Endpoints:

$x = -\frac{1}{6}$: $\sum_{n=1}^{\infty} \frac{6^n \cdot (-\frac{1}{6})^n}{n^4} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ convergent by AST

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} = \sum_{n=1}^{\infty} (-1)^n b_n$, where $b_n = \frac{1}{n^4}$

* $b_{n+1} = \frac{1}{(n+1)^4} \leq \frac{1}{n^4} = b_n$
* $\lim_{n\to\infty} b_n = 0$

OR you can also see that this is absolutely convergent
$\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^4}\right| = \sum_{n=1}^{\infty} \frac{1}{n^4}$ convergent p-series ($p=4$)

$x = \frac{1}{6}$: $\sum_{n=1}^{\infty} \frac{6^n \cdot (\frac{1}{6})^n}{n^4} = \sum_{n=1}^{\infty} \frac{1}{n^4}$ convergent p-series ($p=4$)

=> Radius of convergence: $R = 1/6$
Interval of convergence: $I = \left[-\frac{1}{6}, \frac{1}{6}\right]$