Richard O. Hill

Electricians Need Algebra, Too

The story begins with a thirty-second commercial, aired in spring and summer 2000 on WILX-TV, channel 10, a Lansing-area television station. The commercial, for the International Brotherhood of Electrical Workers (IBEW) and the National Electrical Contractors Association (NECA) stated the following:

> The International Brotherhood of Electrical Workers' apprenticeship program is an opportunity for young men and women to prepare for successful, challenging, and well-paying careers. Apprenticeship with the IBEW provides skills training and the tools you need to build a bright future. If you are at least 17, with a high school diploma or GED, have strong algebra skills, and a desire to join the electrical industry, apply for apprenticeship. . . ."

When I first saw this ad, my reaction was, "What? Algebra skills for electrical workers? Wow!" Through the television station, I contacted Ron Buntin, who is an assistant business manager for the IBEW. He was delighted to discuss the kinds of mathematics problems with which electricians deal and the reasons that they need algebra. He also described the problems faced by the IBEW in finding qualified recruits.

Algebra skills for electrical workers?

Any reasonable person who teaches mathematics knows that a balance is needed among skills, understanding, and problem solving in teaching. Where to put that balance is currently one of the central issues in mathematics education. See, for example, Molnar (2000). This article describes the mathematical needs of electricians and in so doing, indicates where the trainers of electricians strike a balance among skills, understanding, and problem solving.

TRAINING ELECTRICIANS

Buntin put me in contact with Lawrence Hidalgo, the training director for the Lansing Electrical Joint Apprenticeship and Training Committee. I interviewed Hidalgo in September 2000, and most of the following information comes from that interview.

To become an apprentice electrician, a person has to pass a battery of tests that takes about two hours to complete. The tests cover algebra functions, as well as reading comprehension. Those who fail the tests can go through a remedial program (discussed subsequently) and retake the examination in six months. Those who pass are interviewed and ranked and possibly accepted into the internship program when space is available.

An internship is a five-year program that consists of on-the-job training and a full day of classroom studies once every two weeks. An individual who completes the apprenticeship becomes a journeyman electrician. In 2000, a new journeyman electrician earned $26.60 an hour, plus $10,000 a year in a retirement fund, in addition to medical and other fringe benefits. This compensation is good, and contractors are willing to pay it because they know that the skilled-trades unions train people very well. Of course, the union can train only people who have the background to absorb the technical information that they need. Arithmetic and algebra skills, along with reading-comprehension skills, are the skills that otherwise qualified candidates often lack.

MATHEMATICS MATERIALS

We next turn to some materials published by the National Joint Apprenticeship and Training Committee (NJATC) for the electrical industry. The three sample questions from the information booklet for the algebra-and-function test are given in **figure 1.**

In the last question, we see the four ways of describing a function. These ways are emphasized by various reform projects and by many mathematics textbooks. The mathematics section of the actual examination has thirty-three items that candidates must complete in forty-six minutes. As **figure 1** indicates, the algebra demands are really quite modest, essentially at a first-year algebra level. Nevertheless, these arithmetic and algebra demands can be a problem for many candidates.

We next turn to *Mathematics Essential for NJATC Courses* (NJATC 1994). Parts of this book

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1. Consider the following formula: $A = B + 3(4 - C)$. If B equals 5 and C equals 2, what is the value of A?

   (A) 7 (B) 11 (C) 12 (D) 17

2. Consider the following formula: $y = (x + 5)(x - 2)$. Which of the following formulas is equivalent to this one?

   (A) $y = 3x^2 + 9x - 30$
   (B) $y = x^2 + 3x - 10$
   (C) $y = 3x^2 + 3x - 10$
   (D) $y = 3x^2 + 3x - 30$

3. Consider the following equation: $Y = X + 5$. Which of the following choices represent the same relationship as demonstrated in this equation?

   (A)

Y	X
5	10
10	15
15	20
20	25

   (B) Y is equivalent to the sum of a constant C and X, where C equals 5.

   (C) $Y = (X + 20)/4$

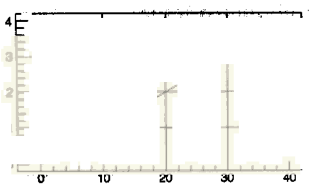
   (D) 

Fig. 1
Sample questions from
Applying and Qualifying for Apprenticeship in the Electrical Industry—Information for Applicants,
NJATC, n.d., pp. 22–23; used with permission

are essentially for the NJATC's remedial course; the chapters of the book are listed in **figure 2.** Much of the book emphasizes skills, and the associated workbook drills the students with a large, but not excessive, number of problems. However, this book and the workbook mix instructions with skill and drill problems, applications, and calculator problems. **Figure 3** gives a few problems from the workbook used in the NJATC's remedial course.

CALCULATORS

All the problems shown in **figure 3** are to be done by hand. I asked Hidalgo why the NJATC wants its apprentices to be able to do these types of computation by hand if they have a calculator. He gave two reasons. First, an electrician might be in the field without a calculator and have to do such calculations. This reason seems a little improbable: I would guess that a professional electrician working in the field on industrial installations that might require computations would have a calculator right next to the wire cutters. However, Hidalgo's second reason is believable. He said that NJATC finds that people better understand the mathematical processes needed to solve problems if they can do them by hand even though they would prefer to use a calculator.

1. Whole Numbers
2. Fractions
3. Decimal Fractions
4. Signed Numbers
5. Powers, Roots (and Logarithms)
6. Units and Measurements
7. Algebra
8. Solving Formulas and Equations
9. Simultaneous Equations
10. Ratio and Proportion
11. Percentage
12. Geometry
13. Trigonometry
14. Vectors
15. Binary, Octal and Hexadecimal Numbers
16. Boolean algebra

Fig. 2
Table of contents of
Mathematics Essential for NJATC Courses

People better understand the mathematical processes needed to solve problems if they can do them by hand

$\begin{array}{r} 14.73 \\ \underline{\times\ .05} \end{array}$ $\begin{array}{r} \$895.89. \\ \underline{\times\ \ 22\%} \end{array}$ $\begin{array}{r} 12.055 \\ \underline{\times 56.297} \end{array}$

$.5 \times 3.2) \times (6.7 \times 2.5) =$ $\$249.49 \times 80\% \times 4\% =$

Linda needs fuses for a motor. If the motor current is 40 amps and the fuses need to be rated at 175% of the motor current, what size fuses will Linda need?

If one case of lamps weighs 22.15 pounds, how much will 3 1/2 cases of lamps weigh?

A 2 in. conduit has a total area of 3.36 square inches. If it cannot be filled over 40%, what area can be filled in a 2 in. conduit?

$1.55\overline{)1824}$ $0.07\overline{)34}$ $0.4\overline{)0.89}$ $8.42\overline{)0.914}$

Alex's paycheck was for \$624.65. If Alex worked for 32.5 hours, what is Alex's rate of pay?

John is mounting a panel on an uneven surface. If he must space the panel 1 3/8 in. off the surface, and the washers he has to use as spacers are .0625 in. thick, how many washers will he need to properly space the panel?

Ohm's law states that the current (I) in amps equals the voltage (E) in volts divided by the resistance (R) in ohms. If you connected a 2-megohm resistor (0.2×10^6 ohms) across a 2.4 kilovolt voltage source (0.2×10^3 volts), what would be the current in amps?

Fig. 3
Sample problems from *NJATC Tech Math Student Workbook*,
used with permission

(2000). However, unions and industry have a legitimate concern about the quality and knowledge of the students we send them. As teachers evaluate potential changes, we must evaluate the core knowledge and skills that students need either to continue their education or to go out into the world. We all need to sit down together and enlarge the conversation.

APPENDIX: CIRCUITRY PROBLEMS

These problems give a sample of the algebra and elementary trigonometry that electricians need when solving electrical circuit problems. The reader is not expected to follow all the physics here but can see the mathematics required. Also, readers should remember that the IBEW does not ask its candidates to come in with all the algebra or trigonometry skills and knowledge necessary to do such problems, but the candidates need to know enough algebra that they can learn and be comfortable with the kind of problems shown here.

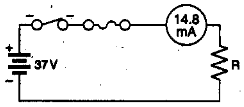
AC circuits are complicated, in that voltage and current are oscillating, as in a sine curve. When they come from a power source, the oscillations are together, or "in phase." But as they go through motors or capacitors, the oscillations of the current can be moved forward or backward relative to the voltage, that is, they become "out of phase." Motors produce *inductance,* which moves the current oscillations forward; capacitors produce *capacitance,* which moves the current oscillations backward. In a problem that deals with balancing out a shop, electricians must keep track of the resistances and phase shifts and then set up the mathematical equations, often manipulating them algebraically to fit the particular installation. The electrician has to understand both the electrical properties and the algebra that describes them. Only then can the electrician do the calculations illustrated in the problems that involve AC circuits.

We must evaluate the core knowledge and skills that students need

The following problems are from *National Electrician Course for Apprentice Inside Wiremen, Second Year Course* (NJATC, 1999).

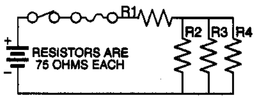
Problems involving direct current (DC) circuits

Find the resistance R in the following circuits. The author's comments are bracketed.



[The apprentice should recognize that this problem is a simple Ohm's law problem and that $E = 37$ and $I = 14.8$ milliamps, both of which need to be converted to amps.]

Answer: $E = I \times R$, $R = E/I$; amps = milliamps/1000, 14.8 mA = 14.8/1000 = 0.0148 A; R = 37/0.0148 = 2500 ohms.



[Each of R_1, R_2, R_3, and R_4 are 75. The apprentice should recognize that three resistances are in parallel, so that the parallel resistance law is used first to find R_{2-3-4}, the resistance of the parallel part. Then R_{2-3-4} and R_1 are in series, so the total R is the sum.]

Answer:

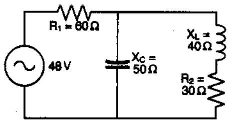
$$\begin{aligned}\frac{1}{R_{2-3-4}} &= \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\\ &= \frac{1}{75} + \frac{1}{75} + \frac{1}{75}\\ &= \frac{3}{75}\\ &= \frac{1}{25},\end{aligned}$$

so $R_{2-3-4} = 25$.

Thus, $R_T = R_1 + R_{2-3-4} = 75 + 25 = 100$ ohms.

Problems involving alternating current (AC) circuits

Using the circuit below, determine the following: (Show work.)



[The diagram shows two branches in parallel and then in series with a resistance. Part (c) is a conceptual question.]

a) What is the impedance of the inductive branch?

Answer:

$$\begin{aligned}Z_{LR} &= \sqrt{R_2^2 + X_L^2}\\ &= \sqrt{30^2 + 40^2}\\ &= 50 \text{ ohms.}\end{aligned}$$

b) The impedance of the parallel circuit is equal to _?_ ohms.

Answer: 79.11.

Work: $I_c = 50/50 = 1$; $I_{LR2} = 50/50 = 1$. The angle of the current through the inductive branch is equal to

$$\begin{aligned}\tan\theta &= \frac{X_L}{R_2}\\ &= 1.33;\\ \theta &= 53.13°.\end{aligned}$$

Thus,

$$\begin{aligned}\sin\theta &= 0.8;\\ \cos\theta &= 0.6;\\ I_{HC} &= I \times \cos\theta\\ &= 0.6 \text{ amps};\\ I_{VC} &= I \times \sin\theta\\ &= 0.8 \text{ amps};\\ H_{C\text{-}BR} &= 0.6\text{ A};\\ V_{C\text{-}BR} &= 1 - 0.8;\\ &= 0.2\text{A};\\ I_{TOT(BR)} &= \sqrt{H_{C\text{-}BR}^2 + V_{C\text{-}BR}^2}\\ &= \sqrt{0.6^2 + 0.2^2}\\ &= \sqrt{0.4}\\ &= 0.632\text{A};\\ Z_{BR} &= 50\text{V}/0.632\text{A}\\ &= 79.11 \text{ ohms.}\end{aligned}$$

a reader be able to see these relationships without algebra?

Furthermore, electricians sometimes have to solve equation (2) for R_{total} or R_1, getting

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \quad + \frac{1}{R_n}}$$

or

$$R_1 = \frac{1}{\frac{1}{R_{total}} - \frac{1}{R_2} - \cdots - \frac{1}{R_n}}.$$

Almost any algebra teacher knows how difficult this algebra can be. In the first equation, what happens to R_{total} if R_1 increases? Could the reader predict this result without algebra? Many students cannot.

At this point, giving more background in electricity and seeing another application with algebra at work might be nice. However, doing so is beyond the scope of this short article; NJATC takes a couple of courses to accomplish this goal for its apprentices. I do, however, give a few sample problems from NJATC materials in the **appendix.** The reader might, but is not expected to, follow the computations in the **appendix.** However, the reader certainly can get the flavor of the algebra required to do the problems. Teachers can use these examples as partial answers to the perennial question, What good is all this stuff for, anyway? The teacher can use these problems as a supplementary project. Students can do research to learn the electrical properties needed to work the problems and write them up. The author of this article would be interested in seeing the results of such a project.

Hidalgo, training director for the Lansing Electrical JATC, notes that candidates who have a reasonable facility with elementary algebra can and do learn the trigonometric and vector information needed to analyze industrial circuits. Of course, Hidalgo has a real advantage with his apprentices. Whenever they become skeptical about the need for all the algebra, he can take them downstairs to the shop, set up the circuits, attach the meters, and show them the algebra in action. More coordination of mathematics and science classes would enable students to see such practical applications as the algebra in electrical circuits around the time that they see the algebra in mathematics class.

I asked Hidalgo for a simple application or example in which an electrician would need computations illustrated in the **appendix.** His real-world example is the following. Suppose that we have a small company that is set up and running well electrically. Then the business expands, machinery is added, and then some more machinery comes on line. The machinery has motors and other electrical devices that change the "phase shift" in the current of the alternating current (AC) circuit of the shop. The electric company then charges the company more than just for the current drawn. An electrician has to come in and analyze the changes, often using algebra to modify the standard equations to fit that installation. Then computations similar to those shown in the **appendix** are made, and capacitors may be installed to put current and voltage back "in phase." (And of course that process saves the company money.)

SKILLS

A reader who examines the NJATC's instructional materials can see that some sections involve pure skill development and other sections involve problem solving. In other fields, such as music and athletics, some time is spent on skill development, and then part of the time is spent playing compositions or playing a game. In mathematics education, however, controversy exists about skills. Some educators are concerned about excessive attention to skills, often using the phrase "drill-to-kill" to represent their concern. Unfortunately, some teachers and educators decry separate skill development, even saying that it is not mathematics. However, skills are important, and educators must seek a balance. To quote from *Principles and Standards for School Mathematics* (NCTM 2000, p. 35),

> Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. On the one hand, computational methods that are over-practiced without understanding are often forgotten or remembered incorrectly (Hiebert 1999; Kamii, Lewis, and Livingston 1993; Hiebert and Lindquist 1990). On the other hand, understanding without fluency can inhibit the problem-solving process. (Thornton 1990).

Rittle-Johnson, Siegler, and Alibali (2001) indicates that conceptual understanding and procedural skills together iteratively improve both.

The real problem is that students heading into different professions need somewhat different skills and understandings. By looking at NJATC materials and elsewhere, as well at college requirements, we can work together to discover what is important to whom.

Skills are important, and educators must seek a balance

CONCLUSION

Skilled trades are seldom thought of as academic endeavors. Thus, learning the types of mathematics that some of the people in them need and how the unions teach the skills is interesting.

So far, much of the discussion about the mathematics to teach has taken place inside the education community. Some of the discussion is strong controversy, sometimes labeled "math wars." The roots of this controversy go back more than a hundred years. See Kilpatrick (1992) and Ravitch

Training materials and Principles and Standards for School Mathematics *make the same basic statement about calculator use*

Calculators—scientific, but not graphing—are used in NJATC materials, and **figure 4** includes NJATC's extremely sensible attitude toward calculator use. **Figure 4** shows that NJATC and the NCTM's *Principles and Standards for School Mathematics* (2000) make the same basic statement about calculator use. Calculators are just not appropriate at all times; at colleges and universities, more and more students turn to their calculators first instead of turning to the mathematics when they are given a problem. Replacing "drill to kill" with "push buttons to kill" has not been advantageous. In other words, the question should not be, Now that we have calculators (or computers), how should we use them? It should be, What mathematics should be taught, and can we use technology to help us teach it? Developing good calculator or computer problems that teach mathematics is surprisingly difficult, but it is being done. The presenters of good technology-enhanced exercises should describe the mathematics that they are trying to teach along with the details of their problems.

NJATC's statement, from *Mathematics Essential for NJATC Courses* (NJATC 1994), p. iii

By following along with these examples, the student will not only apply the mathematical principle(s) being presented, he/she will also be enhancing their calculator proficiency. However, please do not make the mistake of thinking that there is no need to study mathematics. . . . "I'll just use my calculator." If you don't understand the mathematical principles that are involved and how to solve the problem "longhand," you will end up hopelessly lost and confused. The calculator is your friend and often times a very useful tool—it is not a replacement for your mind! Allow your calculator to *expand* your mind, not stifle it.

Statement from *Principles and Standards for School Mathematics* (NCTM 2000, p. 24):

Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions. . . . [Technology] should be used . . . responsibly, with the goal of enriching students' learning of mathematics.

Fig. 4
Statements about calculator use

ALGEBRA CONCEPTS

As the television commercial quoted at the beginning of this article indicates, the IBEW is looking for candidates with good algebra skills. Electricians need to know concepts that just cannot be understood and worked with unless they have some knowledge of algebra. We first give some electrical background.

The most fundamental law of electricity is Ohm's law, $E = IR$. To explain this concept, we use the imperfect analogy that electricity flowing through wire is like water flowing through pipes. The E refers to *electromotive force,* say, from a battery or generator, that corresponds to the amount of force, say, from a pump that is pushing water through a pipe. The I refers to *current,* which is analogous to the amount of water that is flowing. The R refers to *resistance,* which corresponds to a waterwheel that is in the system and that the water is pushing. Of course, both wires and pipes also have resistance, which sometimes has to be considered. In either case, $E = IR$ is plausible. We can express Ohm's law without algebra or solve for current and find that current equals electromotive force divided by resistance without algebra. But it certainly would be more awkward and less understandable than an algebraic representation.

We next discuss the following simple, real-life application. An electrician has to decide the size of the wire to be used in different places. The greater the diameter of the wire, the more current that it can carry (analogous to a pipe, which can carry more water as its diameter becomes larger). But of course, wire with greater diameter costs more. If the wire is too small for the current passing through it, the insulation will fail and cause a short, possibly even a fire. So the electrician needs to determine the amount of current that will flow through the wire. In a simple situation, the electrician knows the voltage, E, in volts, supplied by the power company; knows the resistance, R, in ohms, of the equipment in the circuit or equipment that might be used in the future; and applies Ohm's law, $E = IR$. So in this problem, the electrician knows E and R and uses algebra to compute $I = E/R$ (amps). The electrician then simply picks the smallest wire that can handle this current.

Algebra becomes more crucial for analyzing circuits that have several components. For example, if several resistances are in series, which means one after the other, then the total resistance is the sum

$$R_{\text{total}} = R_1 + R_2 + \cdots + R_n. \tag{1}$$

When resistances are put in parallel, which means that the wire has branches, the total resistance is given by

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}. \tag{2}$$

Here an understanding of algebra is absolutely necessary to understand the relationship. And the relationships can be complicated. The reader should be able to look at this equation and see that when resistances are in parallel, $R_{\text{total}} < R_1$ (or R_2 or . . .). The reader should also see that when more resistances are added in parallel, R_{total} decreases. Would

c) How can the parallel circuit have a greater impedance than the 50 ohms in the capacitive branch?

Answer: The total current needed for the branches is reduced because the inductive current cancels the capacitive current.

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