A Capstone Course for Prospective High School Mathematics Teachers

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Introduction

In 2001, the Conference Board of the Mathematical Sciences (CBMS) published the report *The Mathematical Education of Teachers*, which identifies principles and goals for improving the preparation of teachers, and recommendations for specific mathematics courses for prospective teachers of elementary, middle and high school mathematics. In particular, the report recommends that mathematics departments "support the design, development, and offering of a capstone course sequence for teachers in which conceptual difficulties, fundamental ideas, and techniques of high school mathematics are examined from an advanced standpoint. Such a capstone sequence would be most effectively taught through a collaboration of faculty with primary expertise in mathematics and faculty with primary expertise in mathematics education and experience in high school teaching," ([1], page 39).

This article describes the authors' first attempt at developing such a course at Michigan State University (MSU).

Background and planning

Several conditions at MSU made implementing the recommendation for a capstone course feasible. First, a capstone course has been required for most undergraduate majors for about a decade. To qualify as a capstone course, the course must integrate two or more special fields within a department and include a substantial writing requirement; it is typically taken in a student's senior year. Capstone courses in mathematics all earn 3 credits. Second, the Department of Mathematics includes both mathematicians and mathematics educators. We (Richard Hill, a mathematician, and Sharon Senk, a mathematics educator) were each interested in offering a capstone course for mathematics majors planning to be secondary school teachers. Third, in 2003, the textbook *Mathematics for high school teachers: An advanced perspective* by Usiskin, Peressini, Marchisotto, & Stanley [2], was published that might serve as a core text for a capstone course. We had started reading the text shortly after its release and learned more about it at the AMS-MER workshop on *Excellence in Undergraduate Mathematics: Mathematics for Teachers and Mathematics for Teaching* at Ithaca College in March, 2003. At the workshop, Alice Artzt and Alan Sultan, Queens College CUNY, gave a talk about their experiences using preliminary editions of the text in a course they team-taught. From their presentation and our individual conversations with them, we left the workshop convinced that we should propose to offer a capstone course, based on the book by Usiskin et al, for Fall 2003 at MSU.

Once our proposal to offer such a capstone course was approved, we began to discuss in greater detail how the course would be organized and how we would work as a team. We met weekly in the late spring and several times during the summer of 2003. Syllabi from faculty who had taught from the text at Kennesaw State University were also reviewed.

Course Description

We identified key topics in mathematics needed for teaching in secondary schools: number, algebra/functions, geometry, and trigonometry. With the goal of deepening students' understanding of these topics, we planned to spend 3–4 weeks discussing big ideas for each of these topics. By the end of the course we hoped students would be better prepared than they were at the beginning to describe connections in these areas of mathematics and to figure things out on their own.

As planned, we chose *Mathematics for high school teachers: An advanced perspective* as the textbook. The text's content is rooted in the core content of high school mathematics: numbers, algebra, geometry, measurement, and data analysis; but it is not simply a review of this mathematics. Rather, the authors examine topics in school and college mathematics from an advanced perspective by examining the following:

- Analyses of alternative definitions, language and approaches to mathematical ideas;
- Extensions and generalizations of familiar theorems;
- Discussions of the historical contexts in which concepts arose and evolved;
- Applications of the mathematics in a wide variety of settings;
- Demonstrations of alternate ways of approaching problems, including ways with and without calculator or computer technology;
- Discussions of relations between topics studied in this course and contemporary high school curricula.

Ultimately, we assigned reading and problems from all sections of Chapters 1. *What is meant by an advanced perspective?* and 2. *Real and Complex Numbers* and some sections of the following chapters: 3. *Functions* , 4. *Equations* , 5. *Integers and Polynomials*, 7. *Congruence*, 8. *Distance and Similarity*, and 9. *Trigonometry*. We also studied additional topics not in the text, including development of methods for solving cubic and quartic polynomial equations, a proof of the Fundamental Theorem of Algebra, a proof that there are only five regular polyhedra, and specific examples of how the concepts we were studying appear in contemporary school mathematics textbooks. Some additional resources used were: *What Is Mathematics?* by Courant and Robbins [3], *The nature and growth of modern mathematics* by Kramer [4], a videotape on the Platonic Solids produced by the Visual Geometry Project [5], and lessons from textbooks from the Connected Mathematics Project and the University of Chicago School Mathematics Project.

The course met twice a week, on Tuesdays and Thursdays for 80 minutes, for 15 weeks, and we (Hill and Senk) met on Mondays and Wednesdays to plan the details of what would be done in the following session. Except for illness, each of us attended class every day. Class time was spent in a variety of ways: lecture, whole class discussion, and having students work individually or in small groups. As we modeled different kinds of teaching strategies, we sometimes pointed them out to the students so they could think about what kinds of strategies are appropriate for different kinds of mathematical content or goals.

We assigned reading and homework problems from the text almost every day and usually collected homework assignments on Thursdays, gave three 40-minute quizzes and a final exam, and assigned two papers. We found out early on that we had very similar grading standards. To grade the homework, we picked a few problems from each assignment, and each of us graded half of the papers. We each graded one-half of the problems on every 40-minute quiz and on the final exam paper. (On one of the quizzes, students were allowed to redo questions they missed on the quiz and we averaged the two grades on that quiz.) On the quizzes and final exam we asked questions designed to evaluate the extent to which students were able to describe connections in mathematics and to figure things out on their own. For the first paper, which was five pages, students were asked to write on one of four projects from the text. For the second paper, which was ten pages, students had to write a conceptual analysis of one of four topics from school mathematics: slope, logarithm, matrix, and ellipse. Students were asked to examine the history and applications of the topic, compare and contrast how high school texts approached the topic, and discuss how the topic extends to content in higher-level college courses they had studied.

Students

In order to take a capstone course in the Department of Mathematics at MSU a student must be majoring in mathematics and have completed the core junior-level courses, namely, linear algebra, algebra, and analysis. In addition, we required that students have a teacher preparation program in the College of Education. The 23 students, who applied for this course, all met these criteria.

By most measures the students had strong backgrounds in mathematics. Many had completed more mathematics than the required prerequisites. Most students had at least a 3.0 GPA overall; one student had a perfect 4.0 GPA. Many students had a much better average GPA than 3.0 in mathematics courses. Most students were taking a methods course simultaneously in the College of Education. Despite the students' relatively strong backgrounds, their mathematical knowledge of many fundamental concepts was fragile. Here are some examples we uncovered in class discussions:

- A problem in the text asks, "Write 3.14159 as *p/q* where *p* and *q* are integers." One student, actually a very good student in the Honors College, asked, "Does this mean "3 + .14159" or "3 x .14159"?
- Another good student said, "I never did believe 0.999... = 1."
- Many students did not know much about trigonometry or complex numbers, having seen very little since high school.
- Generally, students had a very poor feeling for convergence, which they should have had at their fingertips from junior-level analysis.
- Many could not state the Division Algorithm for polynomials or the Fundamental Theorem of Algebra even though these are covered in their junior-level algebra course.

Furthermore, most students had little initial understanding of how topics from the undergraduate curriculum were related to each other or to the mathematics they would have to teach in high school. For instance, even though all students had completed a course in abstract algebra, they had little or no appreciation of relations between the Fundamental Theorem of Algebra, the Factor Theorem, and factoring polynomials.

Generally, the students worked hard, completed homework assignments as asked, participated in class discussions, and asked many questions. Most of the required papers were well-written. In particular, the students' work on the second papers showed that they were able to make many connections between topics from high school and college mathematics, when allowed to work outside of class and to consult references. However, students' performance on the assessments administered during class with a time limit varied quite a bit. The average final grade in the course was 3.0 (on the nose). We gave two 4.0s, one 1.5, and the rest were 2.5, 3.0, or 3.5.

Working as a team

Prior to this experience our only previous collaboration had been serving on the same committee together. We had never even taught the same course at the same time. So, prior to agreeing to work together we asked each other many questions about how we would approach certain issues.

We complemented each other very well. Hill currently directs the Emerging Scholars program at MSU and has a strong interest in the transition from high school to college mathematics. He taught high school mathematics for one year before going to graduate school. He has taught calculus and all the core junior-level mathematics courses. Hence, he knows what mathematical knowledge students should potentially bring to a capstone course. Senk had taught high school mathematics for about a dozen years and had co-directed the curriculum development for Grades 7-12 done by the University of Chicago School Mathematics Project. She has done research on high school students' understanding of algebra and geometry and currently is co-PI of a study examining knowledge for teaching algebra. Hence, she is knowledgeable about contemporary mathematics curricula, pedagogical issues in secondary schools, and research on learning and teaching school mathematics.

The course was a lot of work for both of us. Of course, part of this is because we were developing a new course and neither of us was familiar with the textbook. In addition, we were often surprised about what the students did and did not know, and we had to be light on our feet.

One very big problem in setting up the course was deciding who gets credit for teaching a team-taught course. This problem was not satisfactorily resolved for us last fall, with only one of us getting load credit and the other taking on the capstone course as an extra responsibility. We are continuing to search for a more equitable solution to this problem.

Next steps

Generally we were happy with the capstone course, and the students responded favorably. So, we plan to offer it again next fall. Teaching this course has provoked many interesting discussions around several issues. The first is, "What should be the content of such a capstone course for secondary mathematics teachers?" The textbook we used is a great first step to answering this question. But there are critical issues in school mathematics not dealt with in the textbook. For instance, complex numbers are there, but there is no discussion of things like why the product of the square root of -4 and the square root of -9 is not equal to 6. There is very little discussion of logic or proof, and the role they might have when teaching high school.

Clearly no single textbook or no single course can address all the conceptual difficulties, fundamental ideas, and techniques of high school mathematics from an advanced standpoint. Indeed, *The Mathematical Education of Teachers* recommends a 6-hour capstone course for prospective high school teachers. So, a second issue for us at MSU is where else to find opportunities for students to build a deeper understanding of the mathematics they will have to teach in high school and its connections to the college mathematics curriculum. Two places we intend to examine are the other courses undergraduate mathematics majors are required to take and the mathematics methods course(s) students take in the College of Education.

A project at MSU called Teachers For A New Era (TNE), funded initially by a grant from the Carnegie Foundation but augmented by a number of other grants, is engaging faculty across the university in an examination of teacher education in four subject areas. As part of the TNE work in mathematics we have begun to examine ways in which faculty in the junior-level courses in abstract algebra and linear algebra might help improve students' understanding of the mathematics needed for teaching in high school.

Another part of the TNE project is engaging in a university-wide assessment program. One tentative plan is to examine the growth of mathematical reasoning and proof among prospective teachers at different stages of their studies at MSU. We plan to coordinate our work in the capstone course with those efforts by securing permission from our students next fall to participate in this research.

Next fall we also plan to coordinate the content of the mathematics capstone course with the instructor of the methods course taken by prospective secondary mathematics teachers. Because the text by Usiskin et al is a mathematics book, there is virtually no discussion of pedagogical or curricular issues in it. However, pedagogical and curricular knowledge are related to the content one is teaching. For instance, various definitions of concepts such as trapezoid or rotation appear in school and college mathematics textbooks. Teachers need to know the mathematical implications of any particular definition, such as what theorems

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follow from the definition. Such questions are clearly in the domain of a mathematics class. However, prospective teachers also need to know what difficulties students of different ages might encounter when trying to learn a particular definition and how to address questions from students, parents or colleagues who may have encountered other definitions previously. Should such questions also be addressed in a mathematics capstone course? Or should they be addressed in a methods course? Or in both courses? Before next fall we hope to start to sort out the overlap of pure subject matter knowledge and other types of knowledge needed for teaching with our colleagues who teach in the College of Education.

Questions about what fundamental understanding is crucial for future high school teachers and what would be nice for them to know and understand at what level will all be topics of discussion for quite awhile. Various models of capstone courses and possible textbooks are likely to be developed. We are interested in sharing ideas with others teaching capstone courses in mathematics at other institutions or conducting research related to such courses. We are willing to share materials from our experiences in Fall 2003, and next fall when we hope to develop a more detailed syllabus, we'll be willing to share that with others, as well.

References

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Teaching math for elementary teachers, *continued from page 5*

probably be useful for elementary teachers to know many such examples, especially if these examples could provide children with opportunities to use mathematical concepts that they have been studying. For example, consider the following problem: "You made orange paint by mixing 6 scoops of yellow paint with 2 scoops of red paint. How many scoops of yellow paint will you need if you want to use 5 scoops of red paint to make the same shade of orange paint?" Although we can solve this problem by setting up a proportion, cross-multiplying, and solving the resulting equation, we can also solve the problem just by using multiplication, division, and logical reasoning. If we made 1/2 batch of the original paint we would use 3 scoops of yellow and 1 scoop of red. If we then made 5 batches of this half-batch, we would use 5 x 3 = 15 scoops of yellow and 5 scoops of red paint.

I have argued here that elementary teachers should learn to use mathematical language and definitions carefully and precisely, that they must pay close attention to units, that they should be able to explain the rationale for assertions by drawing on fundamental principles, that they should be able to explain how mathematical concepts apply to solve problems, that they should be able to analyze a line of reasoning critically to determine if it is valid or not, that they should be able to reconcile different ways of thinking about mathematical concepts, and that they should recognize the elementary methods that are a precursor to more advanced or powerful methods. These goals are natural and common in advanced math courses; I have argued here that they can be a natural and useful part of courses for elementary teachers that focus on the mathematics of elementary school.

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