26. Let $A$ be an $m \times n$ matrix.

(b) Show that $A^T A$ and $AA^T$ are both symmetric.

Proof.

$$(A^T A)^T = A^T (A^T)^T \quad \text{(By Algebraic Rule 4 for Transpose)}$$

$$= A^T A. \quad \text{(By Algebraic Rule 1 for Transpose)}$$

By the definition of symmetry, $A^T A$ is symmetric.

$$(AA^T)^T = (A^T)^T A^T \quad \text{(By Algebraic Rule 4 for Transpose)}$$

$$= AA^T. \quad \text{(By Algebraic Rule 1 for Transpose)}$$

By the definition of symmetry, $AA^T$ is symmetric. $\square$