

L^AT_EX Graphics with P_STricks

This presentation is also available online. Please visit my home page and follow the links.

<http://www.math.msu.edu/~hensh>

1. Resources

- (a) ImageMagick is a collection of (free) image manipulation tools. You can find out more by visiting

<http://www.imagemagick.com>

- (b) The L^AT_EX Graphics Companion.

- Paperback: 608 pages
- Publisher: Addison-Wesley Pub Co; 1st edition (April 15, 1997)
- ISBN: 0201854694

- (c) The L^AT_EX Graphics Companion (2nd Edition).

- Paperback: 976 pages
- Publisher: Addison-Wesley Professional; 2 edition (August 12, 2007)
- ISBN: 0321508920

- (d) P_STricks: Graphics and PostScript for T_EX and L^AT_EX.

- Paperback: 912 pages
- Publisher: UIT Cambridge Ltd. (September 1, 2011)
- ISBN: 1906860130

- (e) The P_STricks web site.

<https://www.tug.org/PSTricks>

- (f) PostScript(R) Language Tutorial and Cookbook (also called the “The Blue Book”)

- Paperback: 256 pages
- Publisher: Addison-Wesley Professional (January 1, 1985)
- ISBN: 0201101793

2. P_STricks

(a) We start with some examples.

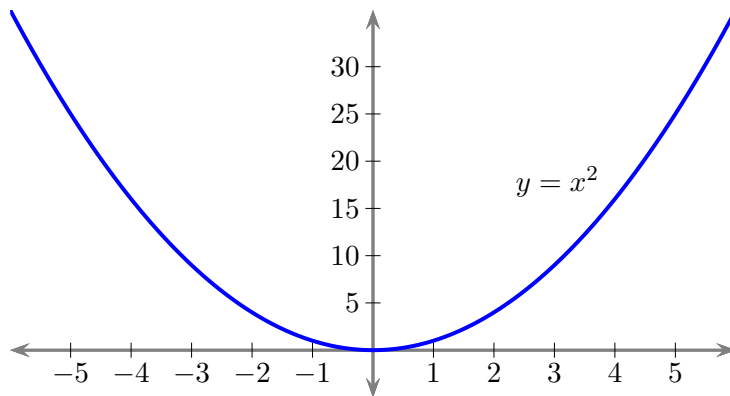


Figure 1: Graphing simple functions.

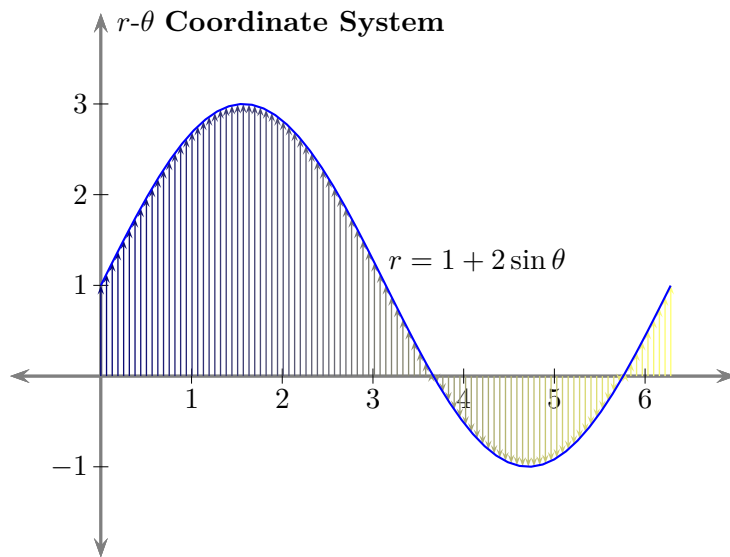


Figure 2: Graphing with some fancy effects.

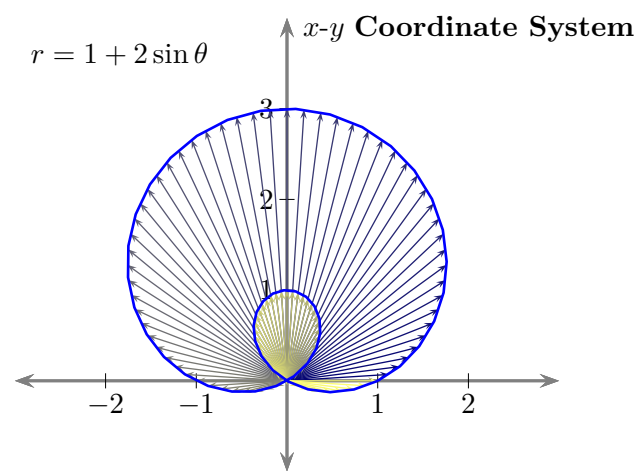


Figure 3: Polar Graphs

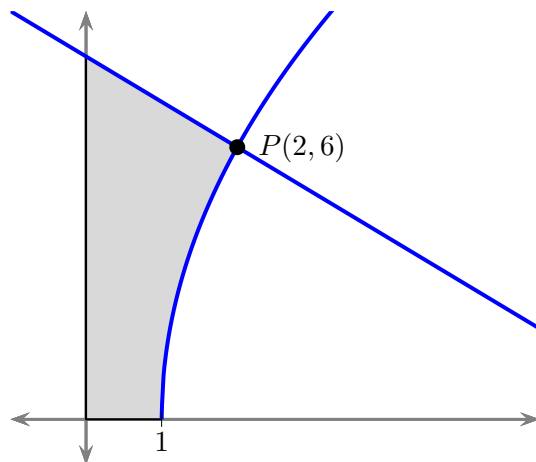


Figure 4: Area between two curves.

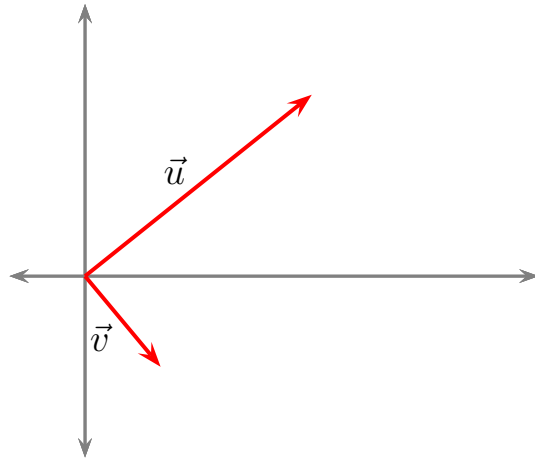


Figure 5: A pair of vectors

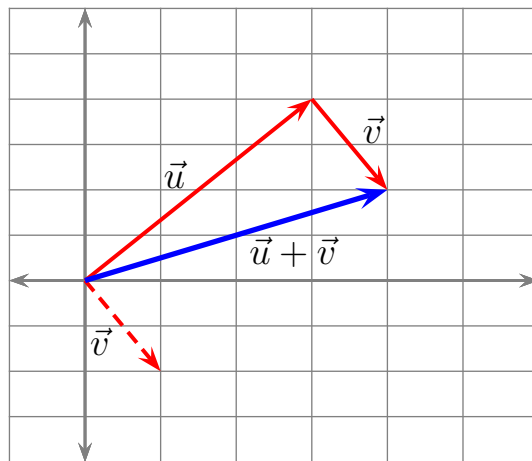


Figure 6: Vector addition with a grid.

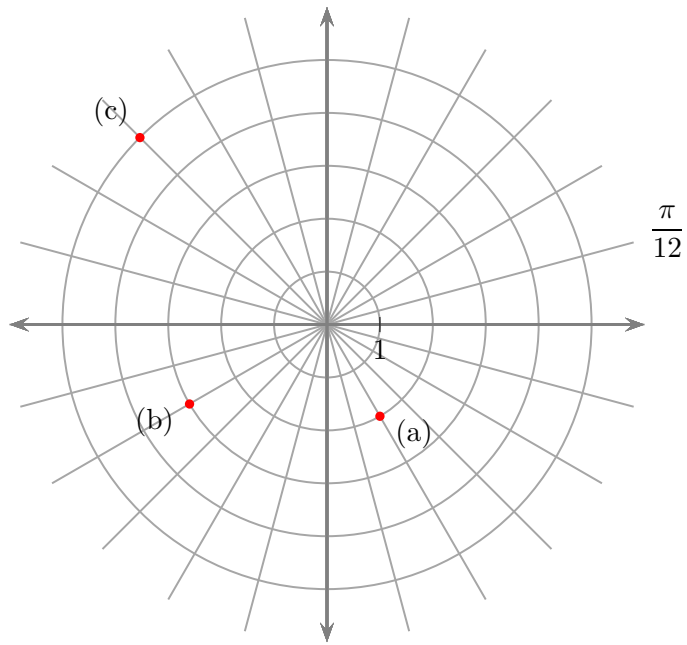


Figure 7: Polar Grid

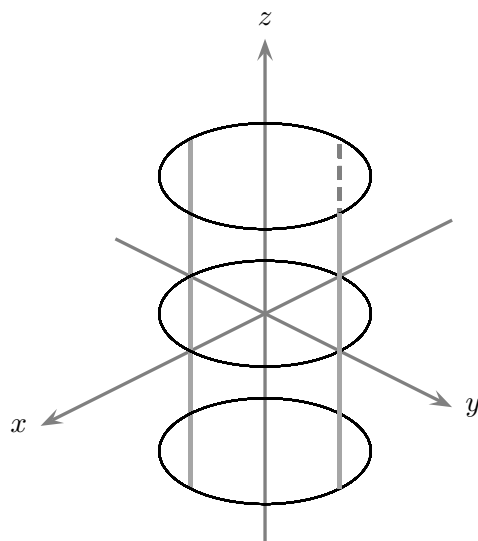


Figure 8: Sketching a cylinder.

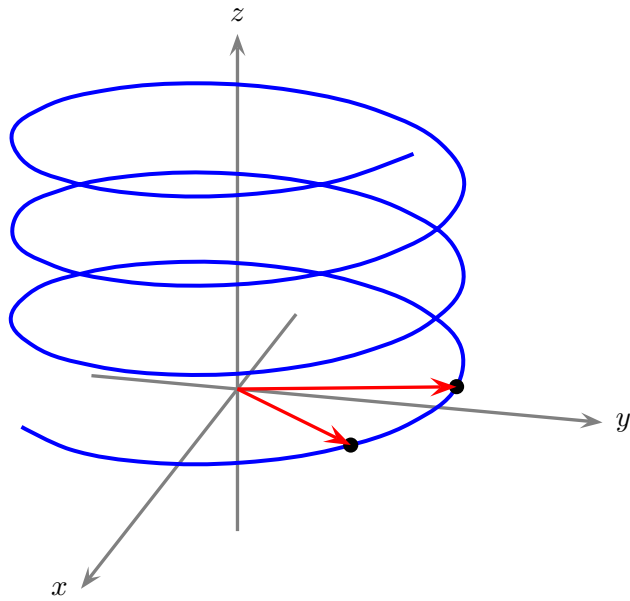


Figure 9: Position Vectors

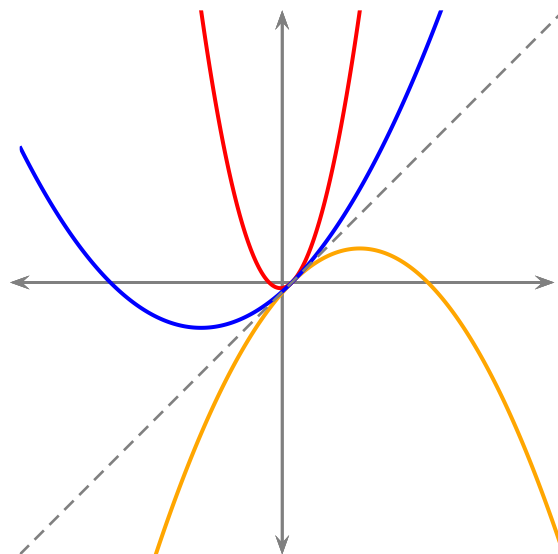


Figure 10: Level Curves

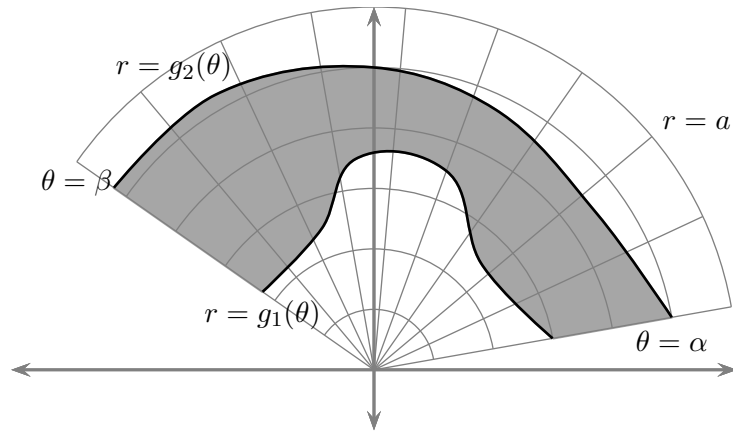


Figure 11: Polar Area

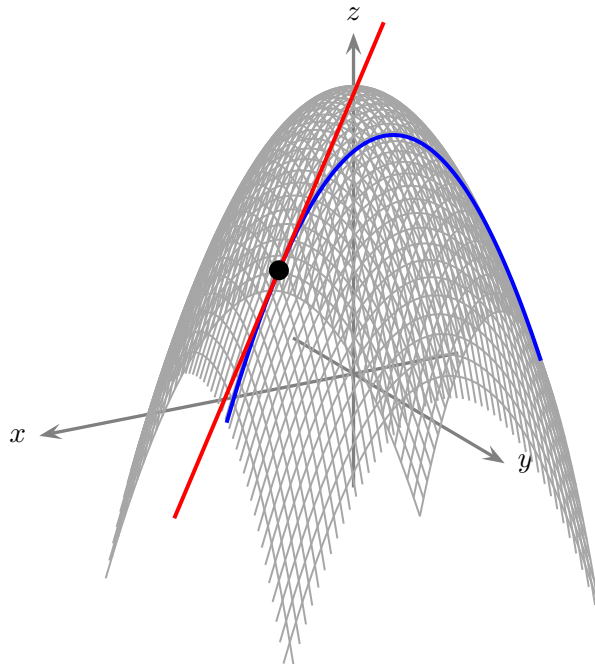


Figure 12: A Tangent Line

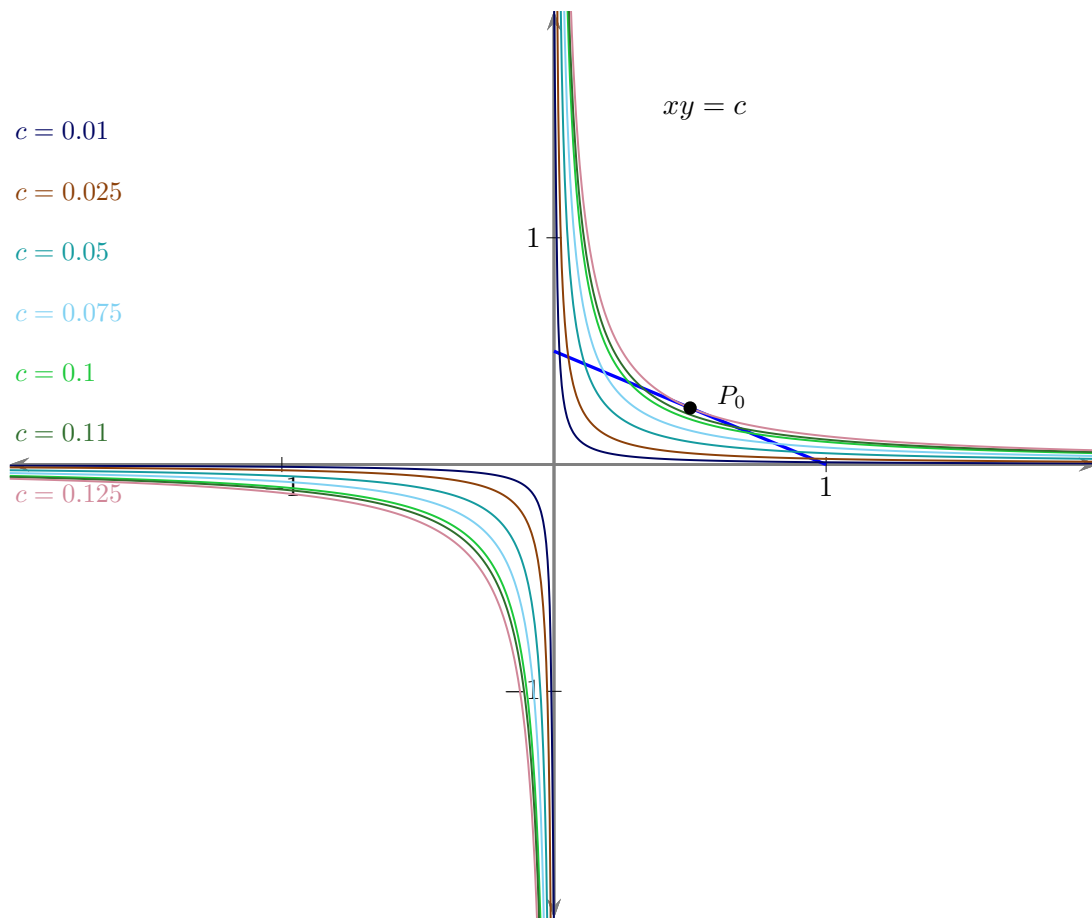


Figure 13: Lagrange Multipliers

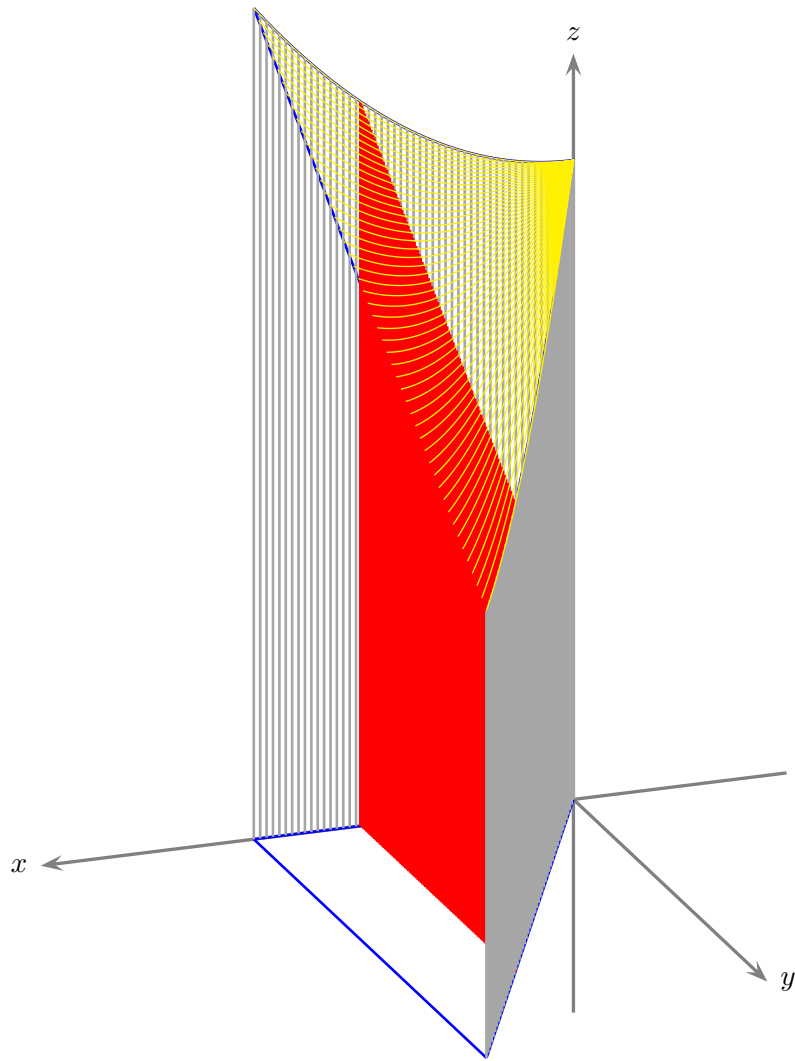


Figure 14: Exposed Solid

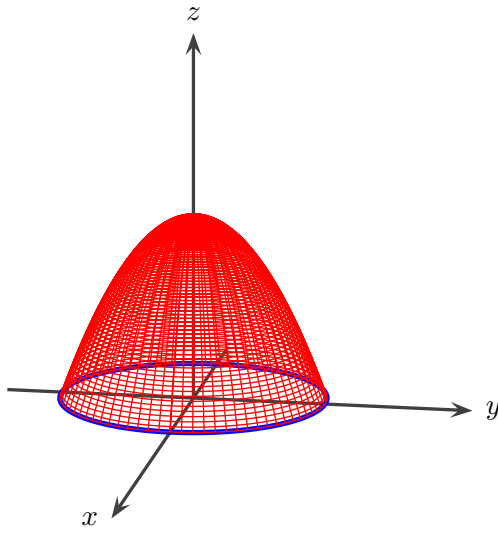


Figure 15: Surface Integrals

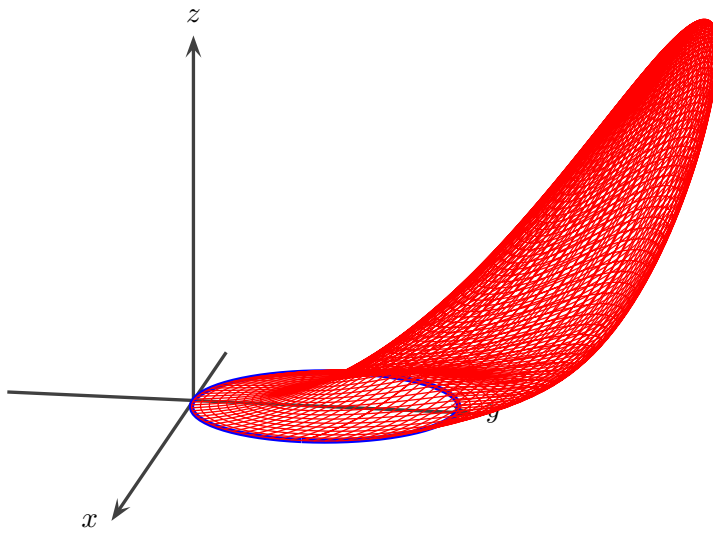


Figure 16: Distorted Surface



Figure 17: Fractals



Figure 18: Lens Effects

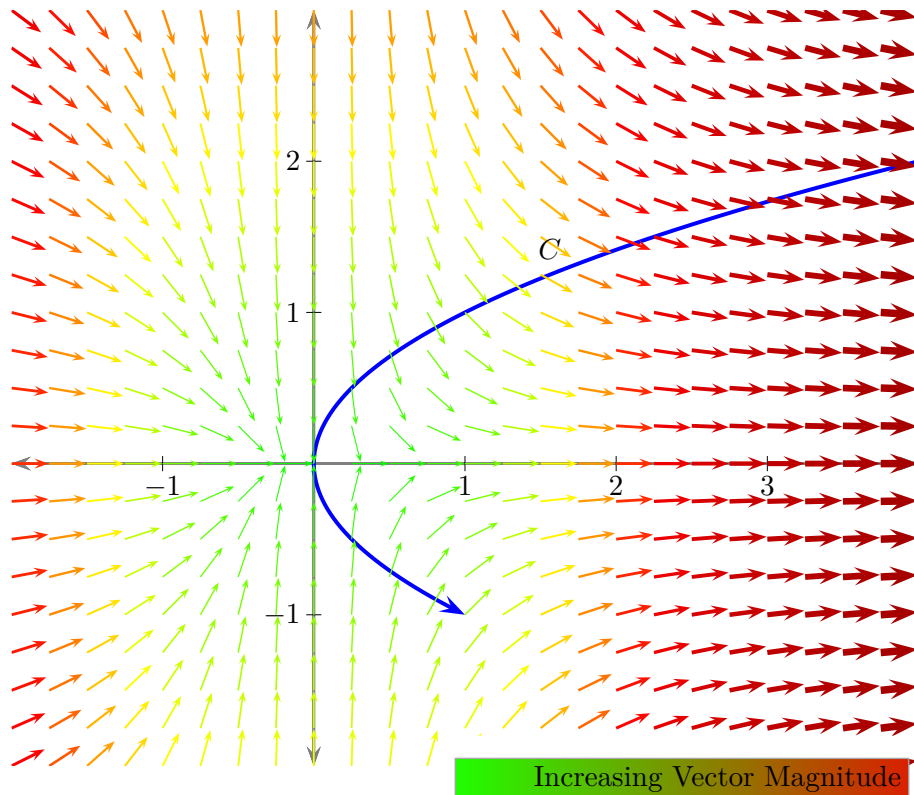


Figure 19: Vector Field

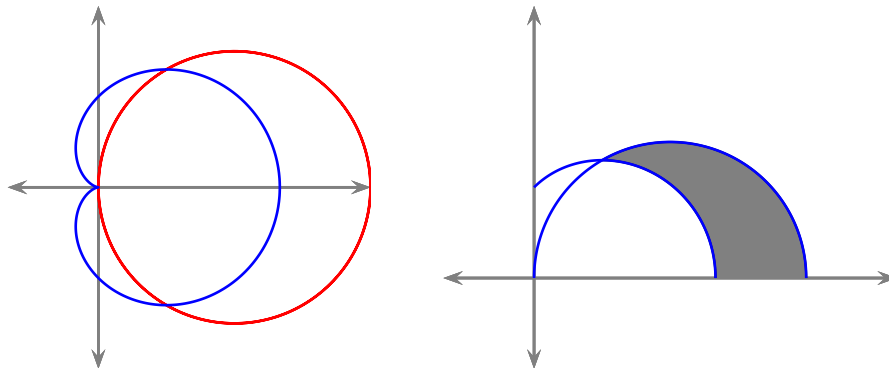
3. PSTricks

To use PSTricks you must include the following lines in the preamble of your document.

```
1 \usepackage{pst-eucl}
2 \usepackage{calc}
3 \usepackage{pst-3dplot}%
4 \usepackage{pst-grad}
5 \usepackage{pst-plot,pst-math,pstricks-add}%
6 \usepackage{pst-all}
7 \RequirePackage{pst-xkey}
```

We should mention that there have been some incompatibilities between the pstcol package (used by PSTricks) and the graphics packages mentioned above.

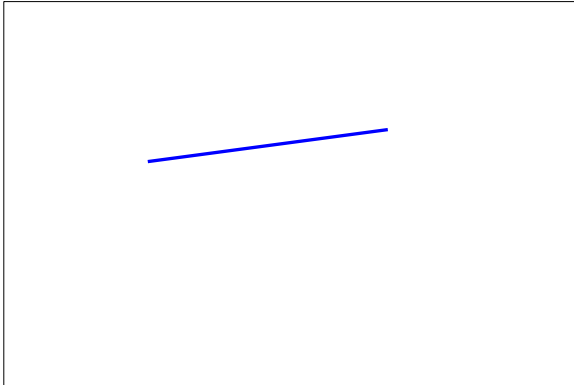
Using colors with PSTricks is similar to what has already been discussed. The real power of the PSTricks package is the ability to *create* graphics using \LaTeX -like syntax.



(a) Preliminaries

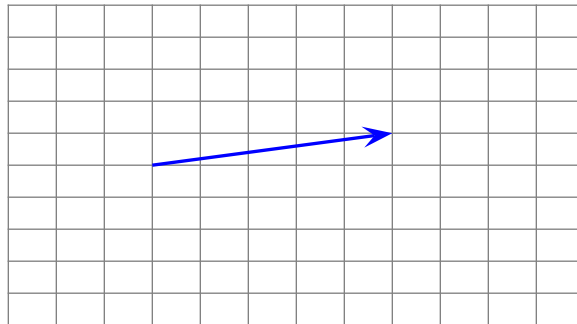
PSTricks provides users with the capability to draw using the familiar syntax of \LaTeX .

```
1 \psline[linecolor=blue,linewidth=1.25pt](-3,1)(2,2)
```



The previous example might be easier to understand if we include more detail in the sketch. Thus

```
1 \showgrid  
2 \psline[linecolor=blue,linewidth=1.25pt,arrowscale=2]{->}(-3,1)(2,2)
```

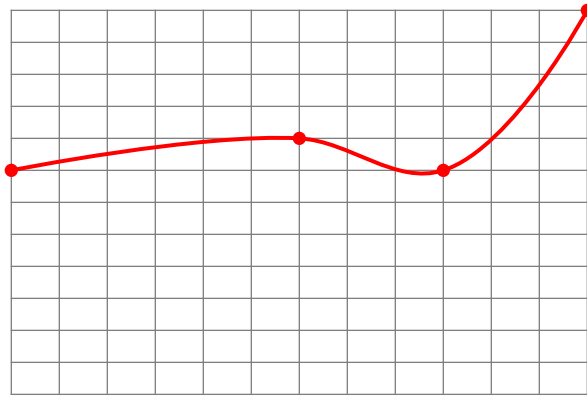


```
1 \newpsobject{showgrid}{psgrid}{%  
2   gridlabels=0pt%  
3   ,griddots=0%  
4   ,gridwidth=0.5pt%  
5   ,gridcolor=gray%  
6   ,subgriddiv=0%  
7   ,subgridwidth=0.25pt%  
8   ,subgridcolor=red}
```

(b) Basic Graphics Objects

Here's a curve. Notice that the points used can be turned on (as shown) or off.

```
1   %\begin{pspicture}(\xmin,\ymin)(\xmax,\ymax)
2       \showgrid
3       \pscurve[linecolor=red,linewidth=1.5pt,showpoints=true]%
4           (\xmin,1)(0,2)(3,1)(\xmax,\ymax)
5   %\end{pspicture}
```



where the values `\xmin`, `\ymin`, etc. have been defined previously as

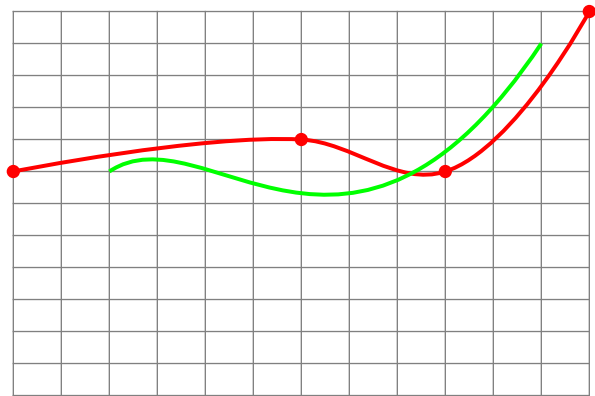
```
1   \def\xmin{-6}\def\xmax{6}
2   \def\ymin{-6}\def\ymax{6}
```

We begin by setting the default unit(s) in PSTricks using the command `\psset{unit=1cm}` . This is actually the default value.

```

1      %\begin{pspicture}(\xmin,\ymin)(\xmax,\ymax)
2          \showgrid
3          \pscurve[linecolor=red,linewidth=1.5pt,showpoints=true]
4              (\xmin,1)(0,2)(3,1)(\xmax,\ymax)
5          \psbezier[style=myCurveStyle,linecolor=green]{-}%
6              (-4,1)(-2,3)(1,-4)(5,5)
7      %\end{pspicture}

```

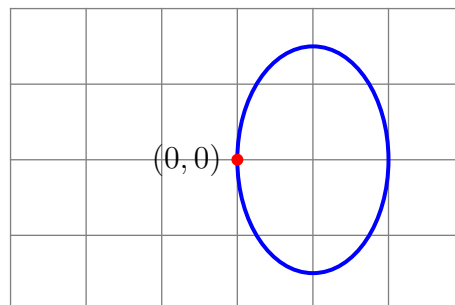


There are built-in shapes

```

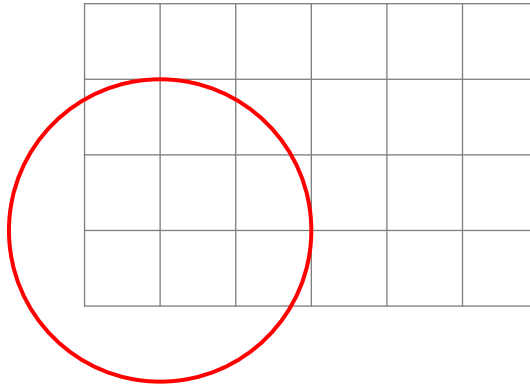
1      %\begin{pspicture}(\xmin,\ymin)(\xmax,\ymax)
2          \showgrid
3          \psellipse[linecolor=blue,linewidth=1.5pt]
4              (1,0)(1,1.5)
5          \psdots[linecolor=red,linewidth=1.25pt](0,0)
6          \SpecialCoor
7              \uput{6pt}[180](0,0){$(0,0)$}
8          \NormalCoor
9      %\end{pspicture}

```



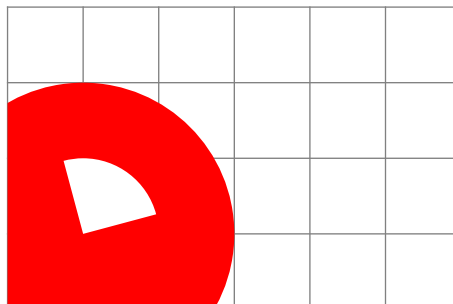
Here is a circle centered at $(-2, -1)$ of radius 2.

```
1   %\begin{pspicture}(\xmin,\ymin)(\xmax,\ymax)
2       \showgrid
3       \pscircle[linecolor=red,linewidth=1.5pt]
4           (-2,-1){2}
5   %\end{pspicture}
```



Here is the same object filled-in and clipped.

```
1   %\begin{pspicture}(\xmin,\ymin)(\xmax,\ymax)
2       \showgrid
3       \psclip{\myframe(\xmin,\ymin)(\xmax,\ymax)}
4       \pscircle*[linecolor=red,linewidth=1.5pt]
5           (-2,-1){2}
6       \endpsclip
7       \pswedge*[linecolor=white](-2,-1){1}{15}{105}
8   %\end{pspicture}
```

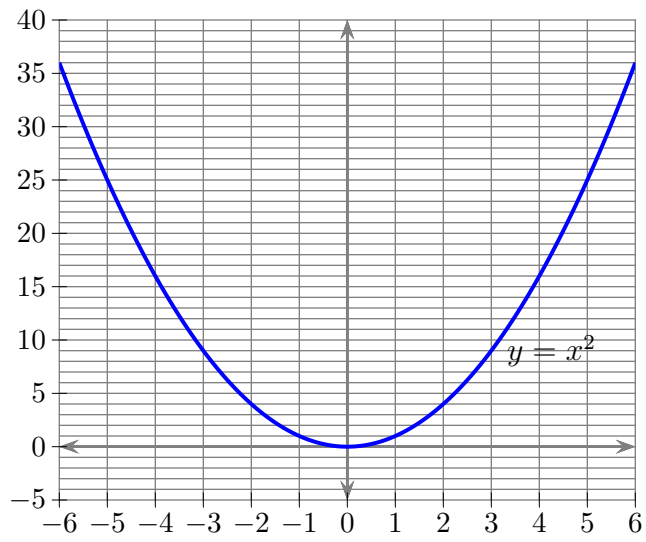


Finally, we plot some functions. To do this we'll use some custom macros that give the user better control over the coordinate system.

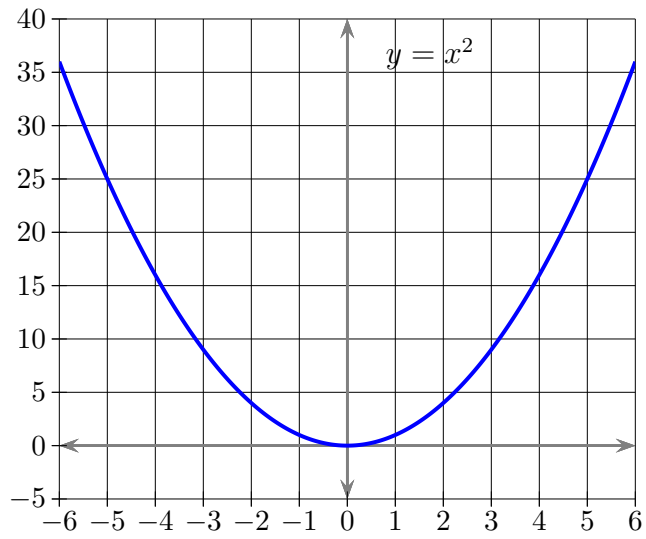
```

1      \begin{pspicture}(\xmin,\ymin)(\xmax,\ymax)
2          \showgrid
3          \pstVerb{%
4              /f@ {dup mul} def % x^2
5          }
6          %%%%%%%%%%%
7          %% Axes and Ticks %%
8          %%%%%%%%%%%
9          \myaxes{<->}(0,0)(\xmin,\ymin)(\xmax,\ymax)
10         \xTickMarks{\xmin}{\xmax}{1}
11         \yTickMarks{\ymin}{\ymax}{5}
12         %%%%%%%%%%%
13         %% Graphical Objects %%
14         %%%%%%%%%%%
15         \psclip{\myframe(\xmin,\ymin)(\xmax,\ymax)}
16             \psplot[style=myPlotStyle]
17                 {\xmin}{\xmax}{x f@}
18         \endpsclip
19         %%%%%%%%%%%
20         %% Labels %%
21         %%%%%%%%%%%
22         \SpecialCoor
23             \uput{6pt}[0](!3 dup f@){$y=x^2$}
24         \NormalCoor
25     \end{pspicture}

```

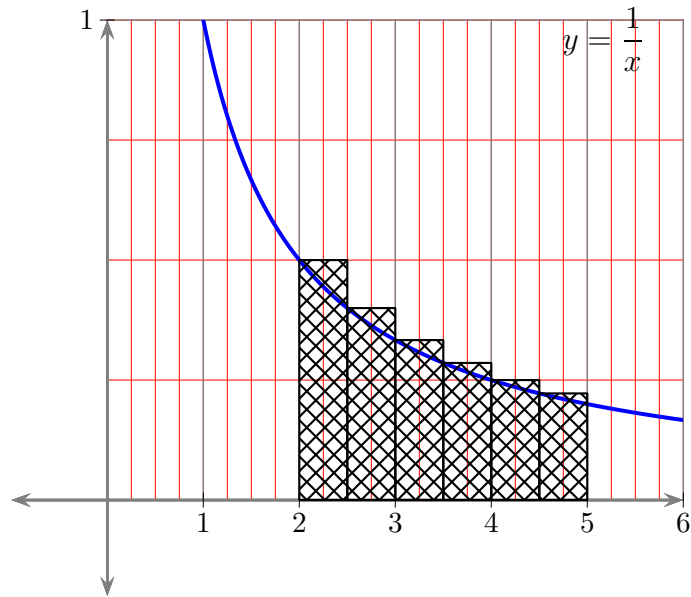


And again, using better grid controls.



Here's something useful for integration theory. Use the sketch to estimate the integral below.

$$\int_2^5 \frac{1}{x} dx$$



Here is the code

```
1 \def\xmin{0}\def\xmax{6}
2 \def\ymin{0}\def\ymax{1}
3 \def\dommin{\xmin}\def\dommax{\xmax}
4 \VR{3in}{2.5in}
5
6 \newpsobject{newgrid}{psgrid}{%
7   gridlabels=0pt%
8   ,griddots=0%
9   ,gridwidth=0.5pt%
10  ,gridcolor=gray%
11  ,subgriddiv=4%
12  ,subgridwidth=0.25pt%
13  ,subgridcolor=red}
14 \begin{pspicture}(\xmin,\ymin)(\xmax,\ymax)
15 \newgrid
16   %%%%%%%%%%%
17   %% Axes and Ticks %%
18   %%%%%%%%%%%
19   \SpecialCoor
20   %% Labels go here
21   \rput[lr](!\xmax\space\xmax\space\xmin\space sub 15 div sub
22   \ymax\space\ymax\space\ymin\space sub 25 div sub){$y=\dfrac{1}{x}$}
23   \NormalCoor
24   \myaxes{<->}(0,0)(\xmin,\ymin)(\xmax,\ymax)
25   %% Change these as needed. #1 - start, #2 - end, #3 - increment
26   \xTickMarks{\xmin}{\xmax}{1}
27   \yTickMarks{\ymin}{\ymax}{1}
28   \psclip{\psframe(\xmin,\ymin)(\xmax,\ymax)}
29   %% Graphing directives go here, e.g.,
30   \psplot[style=myPlotStyle]
31     {0.1}{\xmax}{1 x div}
32   \endpsclip
33   \SpecialCoor
34   \psline[fillstyle=crosshatch]{-}(3,0)(3.5,0)(!3.5 1 3
35   div)(!3 1 3 div)(3,0)
36   \psline[fillstyle=crosshatch]{-}(3.5,0)(4,0)(!4 1 3.5
37   div)(!3.5 1 3.5 div)(3.5,0)
38   \psline[fillstyle=crosshatch]{-}(4,0)(4.5,0)(!4.5 1 4
39   div)(!4 1 4 div)(4,0)
40   \psline[fillstyle=crosshatch]{-}(4.5,0)(5,0)(!5 1 4.5
41   div)(!4.5 1 4.5 div)(4.5,0)
42   \NormalCoor
43 \end{pspicture}
```

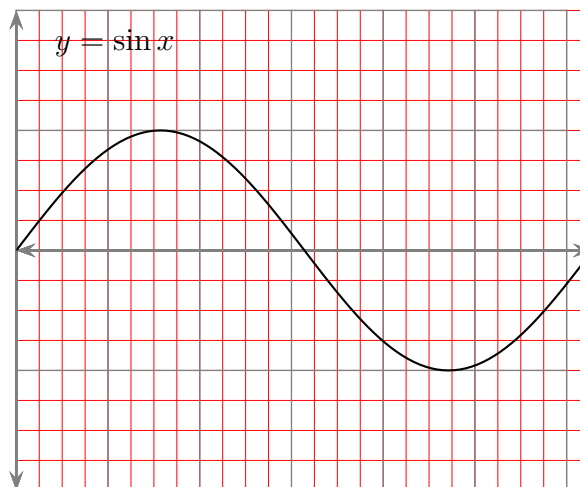
(c) Plotting Data from a File

Suppose that you wish to plot the following data.

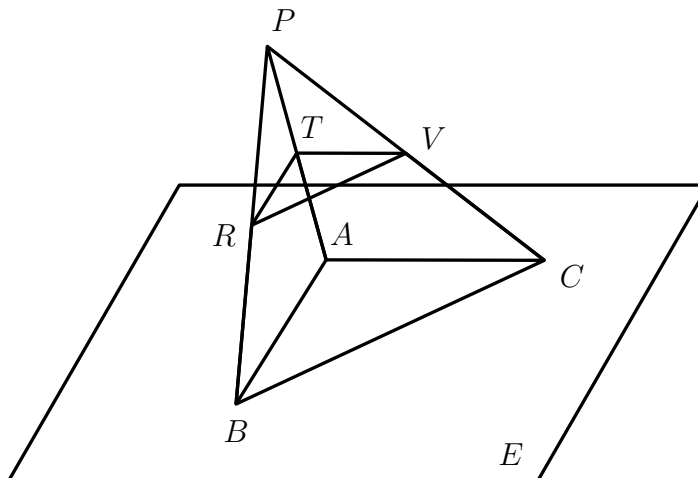
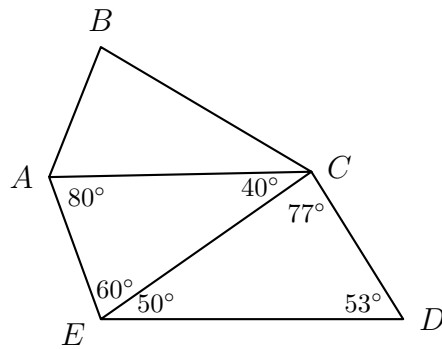
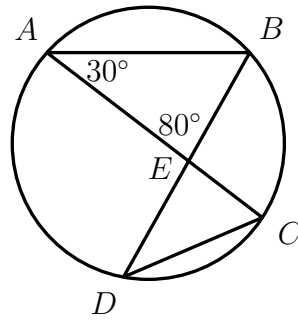
0,	0
0.0628,	0.06279
0.1256,	0.12533
...	...

The following code does the trick.

```
1  %\begin{pspicture}(\xmin,\ymin)(\xmax,\ymax)
2  \showgrid
3  %% Axes and Ticks %%
4  \myaxes{<->}(0,0)(\xmin,\ymin)(\xmax,\ymax)
5  %% Graphical Objects %%
6  \psclip{\myframe(\xmin,\ymin)(\xmax,\ymax)}
7    \fileplot{plotData.txt}
8  \endpsclip
9  \SpecialCoor
10   \rput[lt](!\xmax\space\xmin\space sub 15 div
11   \ymax\space\ymax\space\ymin\space sub 25 div sub){$y=\sin x$}
12  \NormalCoor
13  %\end{pspicture}
```

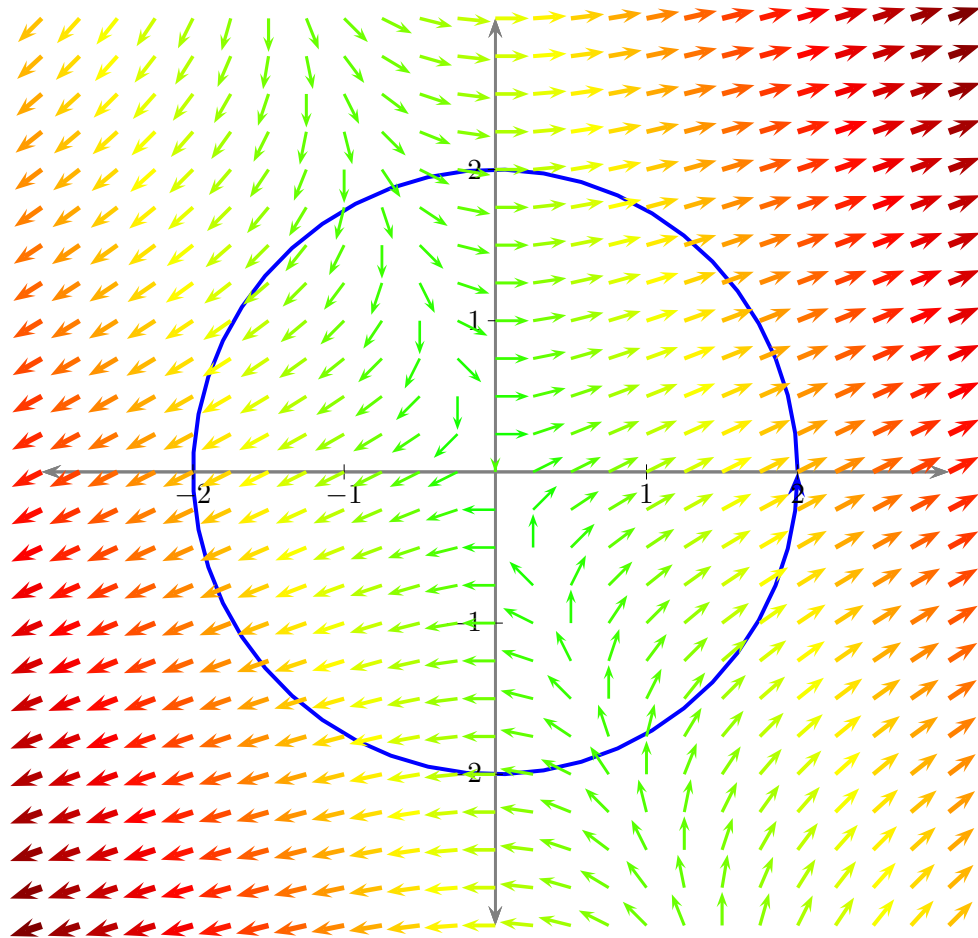


4. Several examples from geometry.

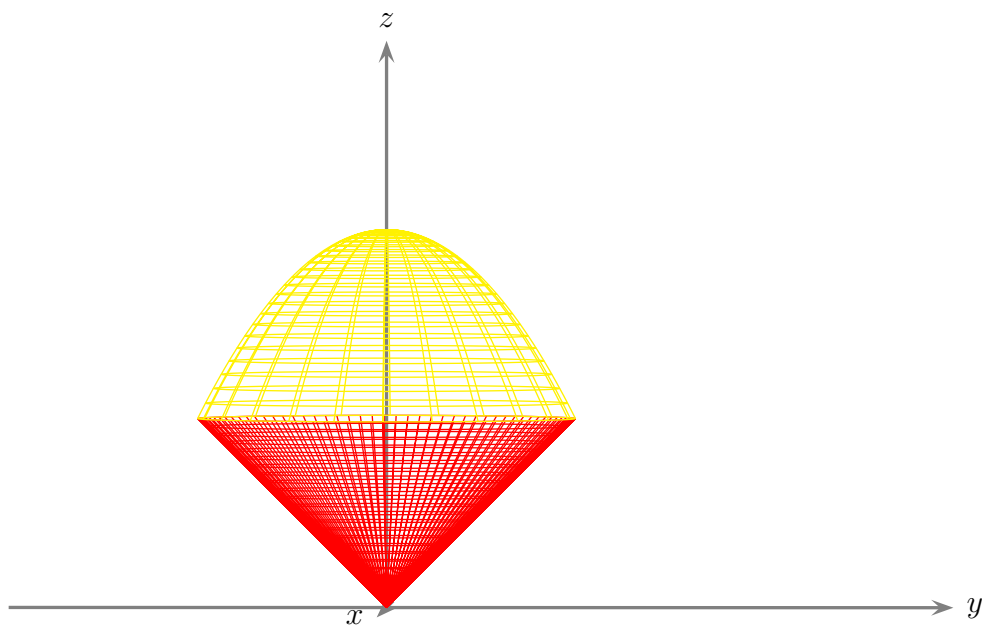
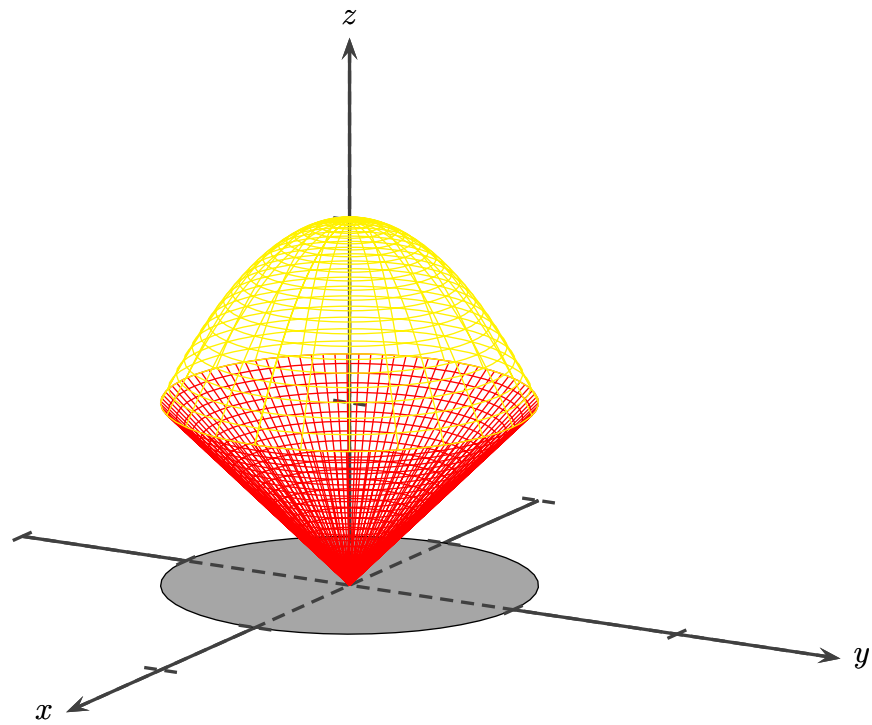


5. A few exotic tricks.

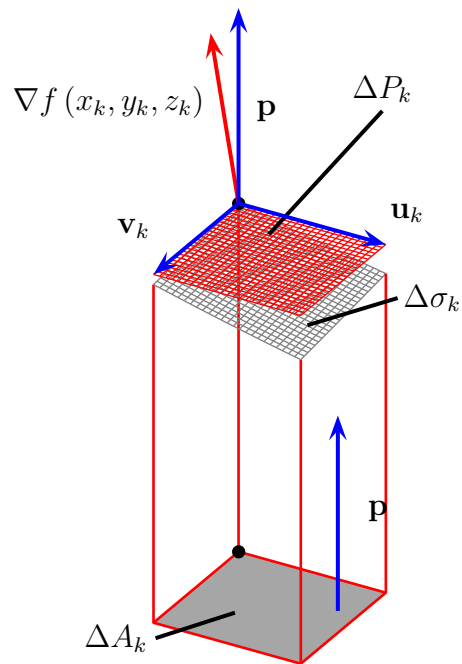
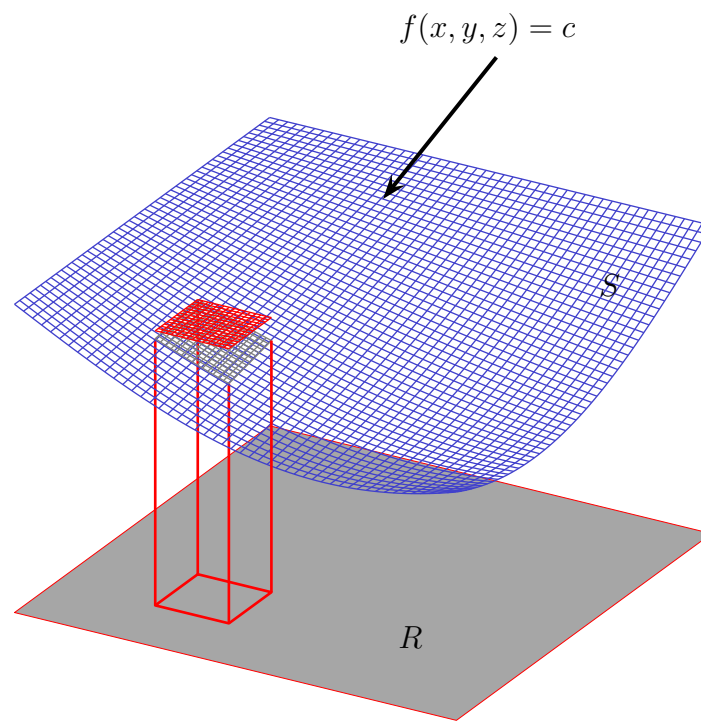
(a) A vector field.



(b) An ice-cream cone.



(c) A level surface.



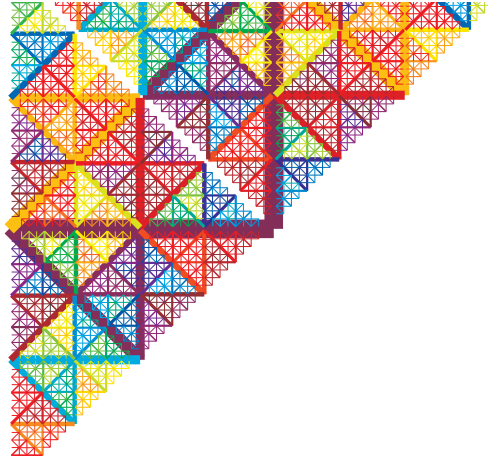


Figure 20: Fractals - Sierpinski Triangle

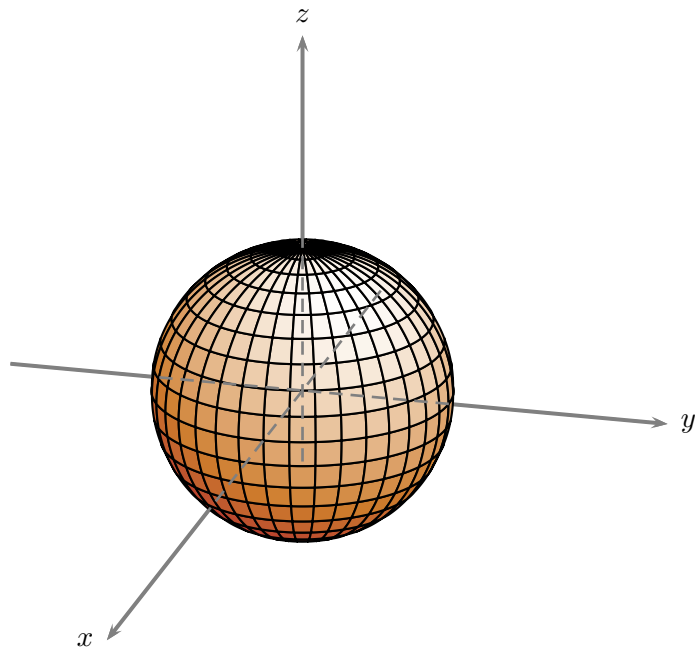


Figure 21: Sphere

The equation of a sphere with radius r centered at the origin is

$$x^2 + y^2 + z^2 = r^2$$

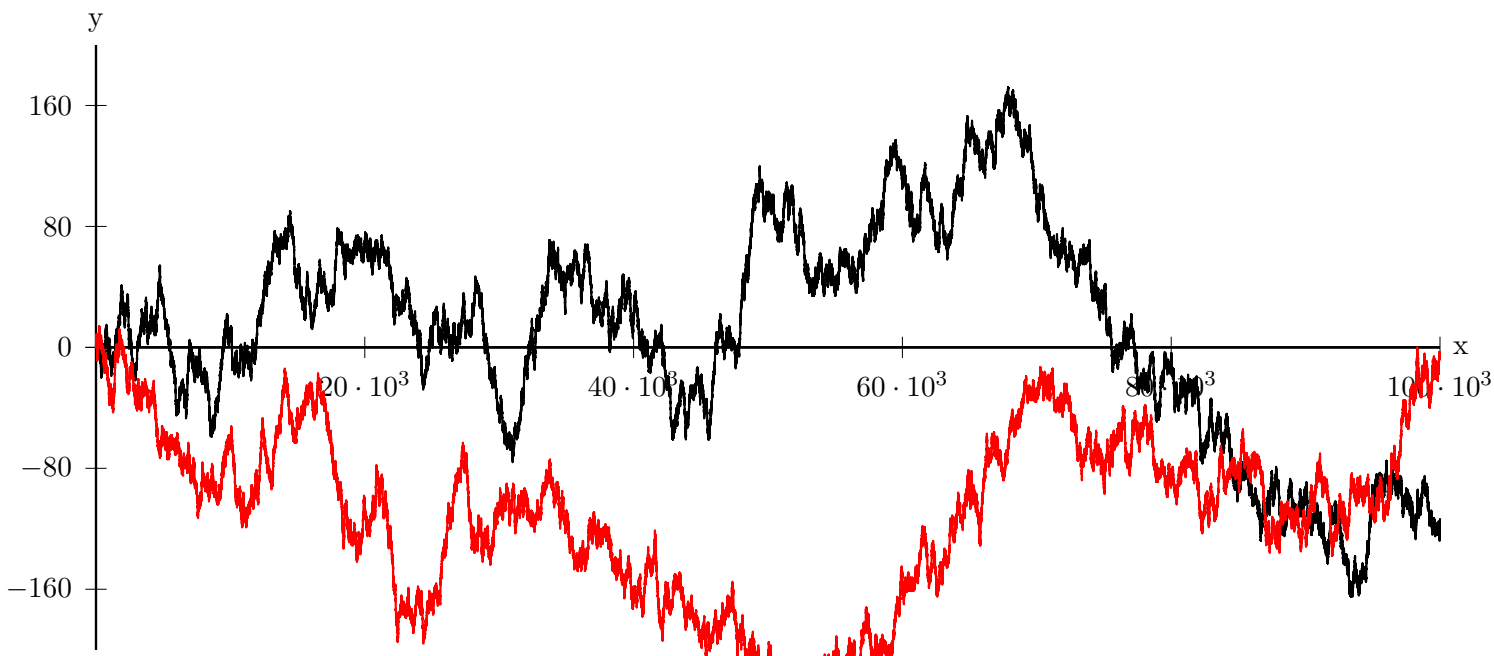


Figure 22: Brownian Motion

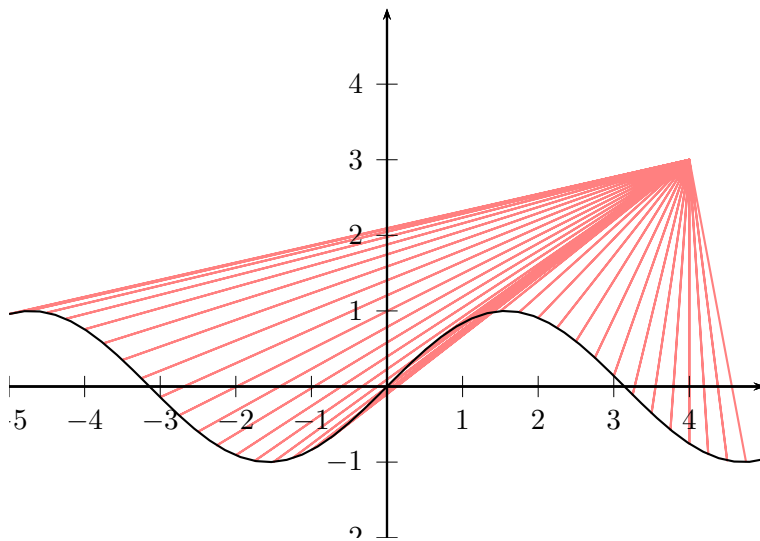


Figure 23: Not Sure What to Call This

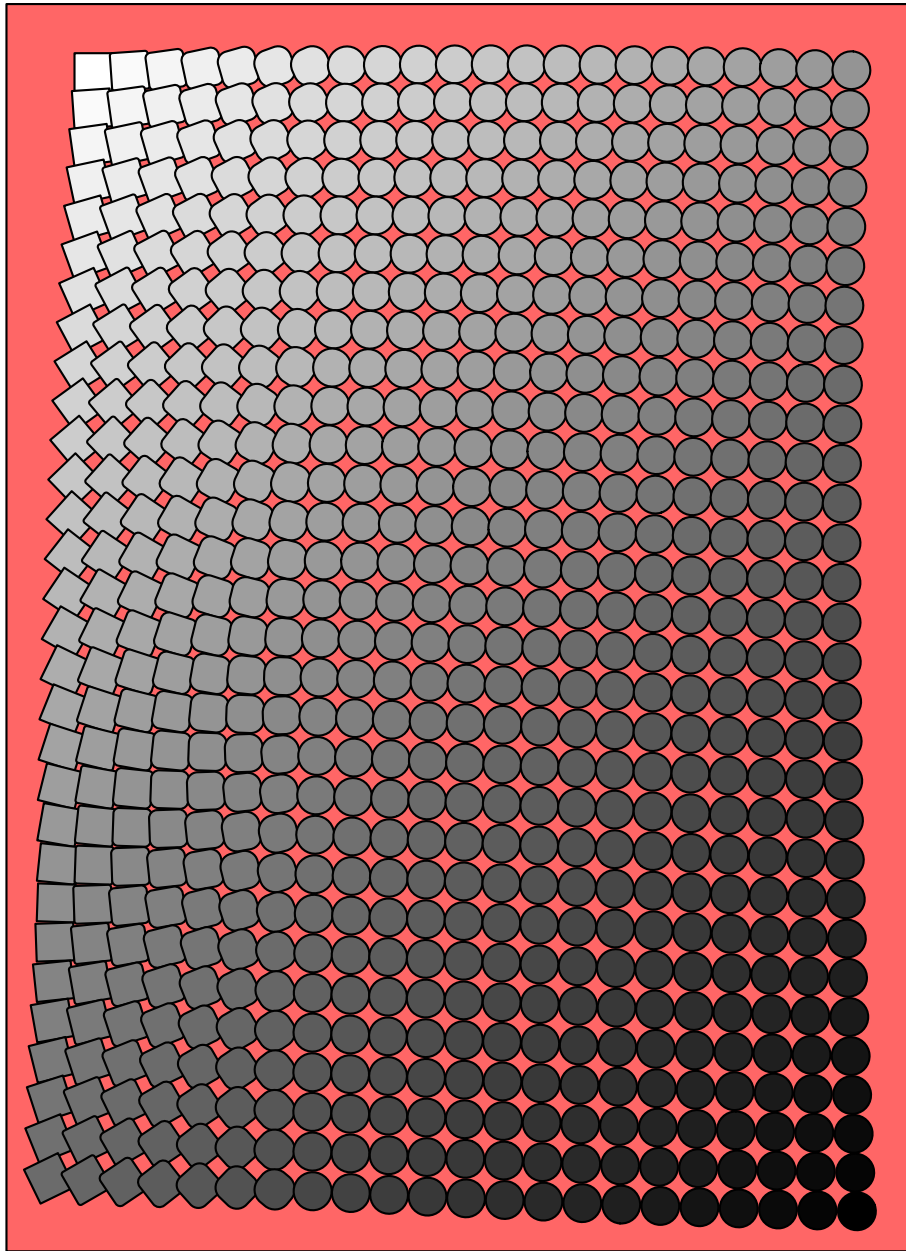


Figure 24: Tessellations?

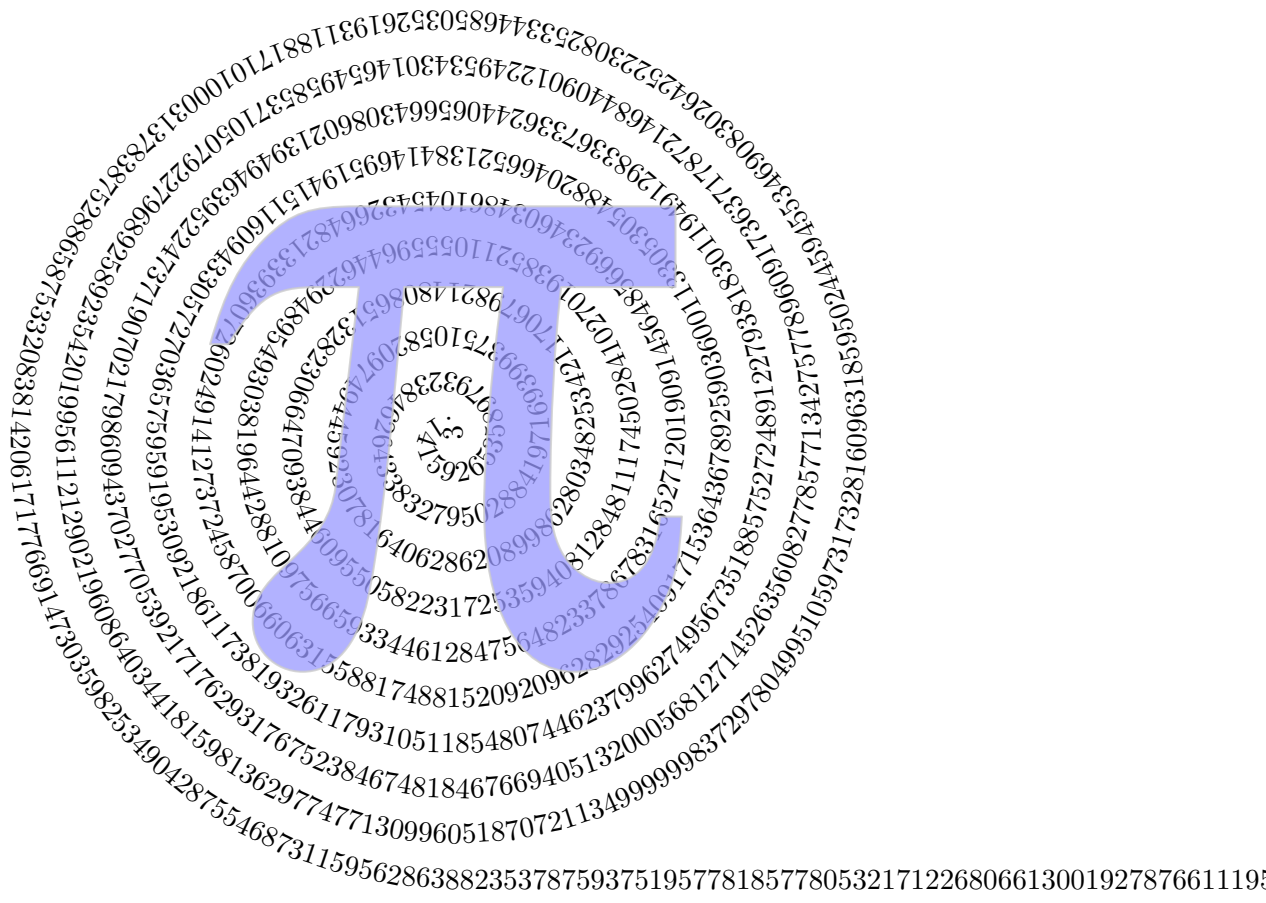
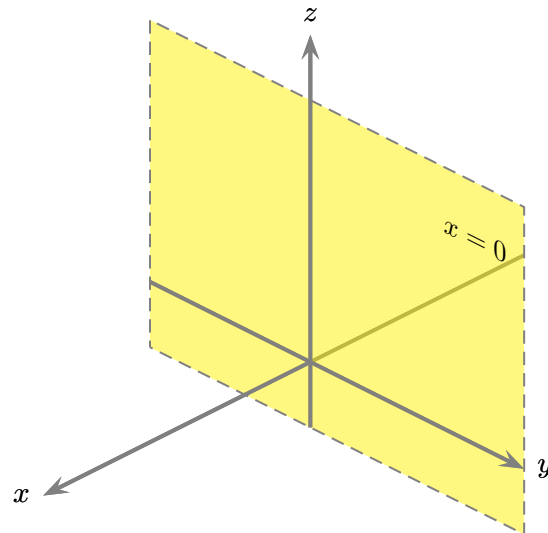


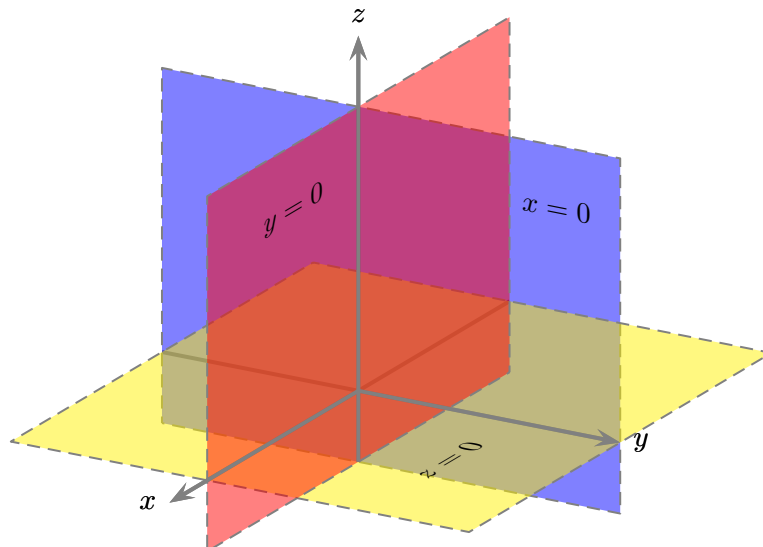
Figure 25: π Spiral

Transparency Effects

The sketch below shows the infinite plane $x = 0$.

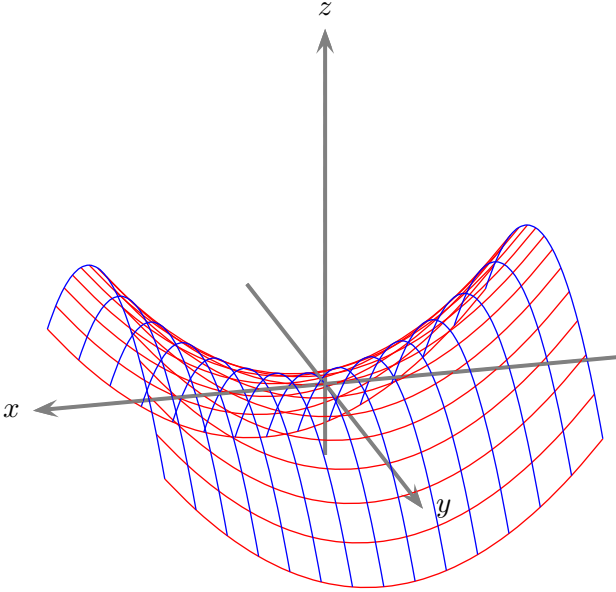
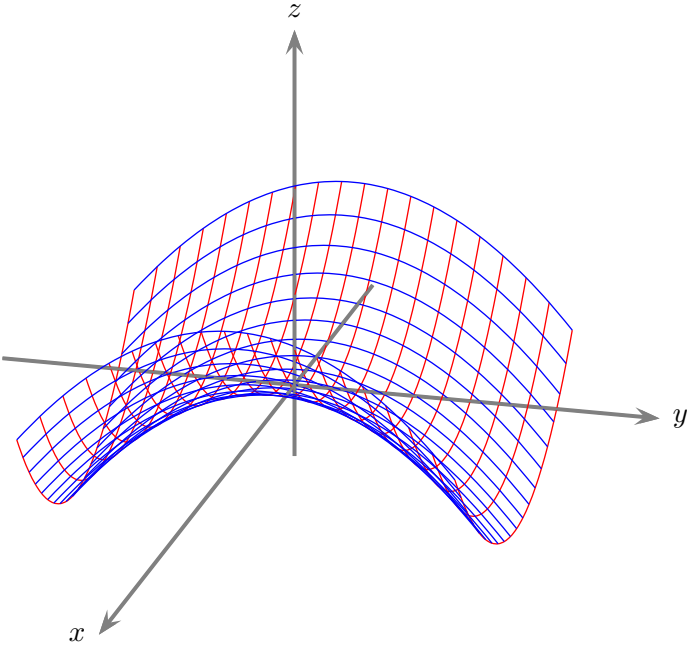


In a similar manner one can sketch the graphs of the equations $y = 0$ and $z = 0$.



Notice that these three planes break up three-space into eight *octants*. The **first octant** coincides with positive x, y and z -coordinates and is three-dimensional analogue of quadrant I in the plane.

A Saddle Point



Example 1. A Familiar Curve

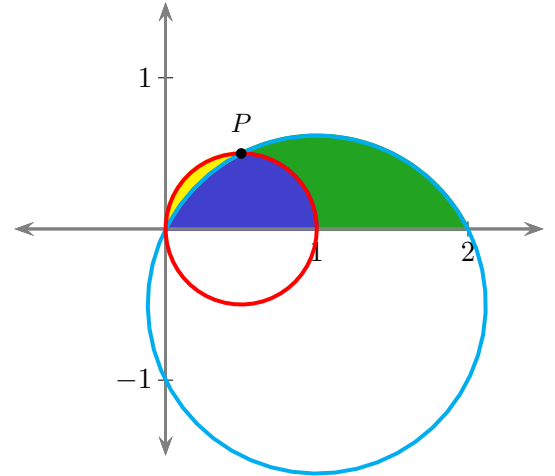
Find the areas of the shaded regions.

$$r_1 = 2 \cos \theta - \sin \theta$$

$$r_2 = \cos \theta$$

We first need to find the polar coordinates of point of intersection, P . Setting $r_1 = r_2$ and solving we see that $\theta = \pi/4$. It follows that the area of green portion of the shaded region is given by

$$A_G = \frac{1}{2} \int_0^{\pi/4} (r_1^2 - r_2^2) d\theta$$



It follows that

$$\begin{aligned}
 A_G &= \frac{1}{2} \int_0^{\pi/4} \sin^2 \theta + 3 \cos^2 \theta - 4 \sin \theta \cos \theta d\theta \\
 &= \frac{1}{2} \int_0^{\pi/4} 1 + 2 \cos^2 \theta - 2 \sin 2\theta d\theta \\
 &= \frac{1}{2} \int_0^{\pi/4} 2 + \cos 2\theta - 2 \sin 2\theta d\theta \\
 &= \frac{1}{4} (4\theta + \sin 2\theta + 2 \cos 2\theta) \Big|_0^{\pi/4} \\
 &= \frac{1}{4} \{(\pi + 1 + 0) - (0 + 0 + 2)\} = \frac{\pi - 1}{4}
 \end{aligned}$$

Notice that

$$A_G + A_B = \frac{1}{2} \int_0^{\arctan 2} r_1^2 d\theta$$

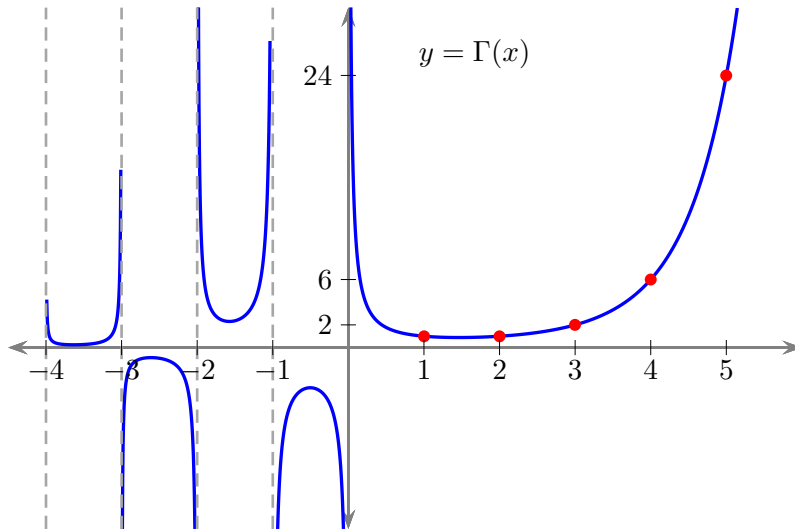
(Compare this last equation to the gas tank problem.)

Finally, can you find A_Y ?

Definition. The Gamma Function

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \tag{1}$$

Here the (improper) integral converges absolutely for all $x \in \mathbb{R}$ except for the non-positive integers. In fact, the Gamma function can be extended throughout the complex plane (again, except for the non-positive integers).



Observe that

$$\begin{aligned} \Gamma(1) &= \int_0^{\infty} t^0 e^{-t} dt \\ &= \frac{-1}{e^t} \Big|_0^{\infty} = 0 - (-1) = 1 \end{aligned}$$

and for positive integers n , integration by parts yields the recursive relation

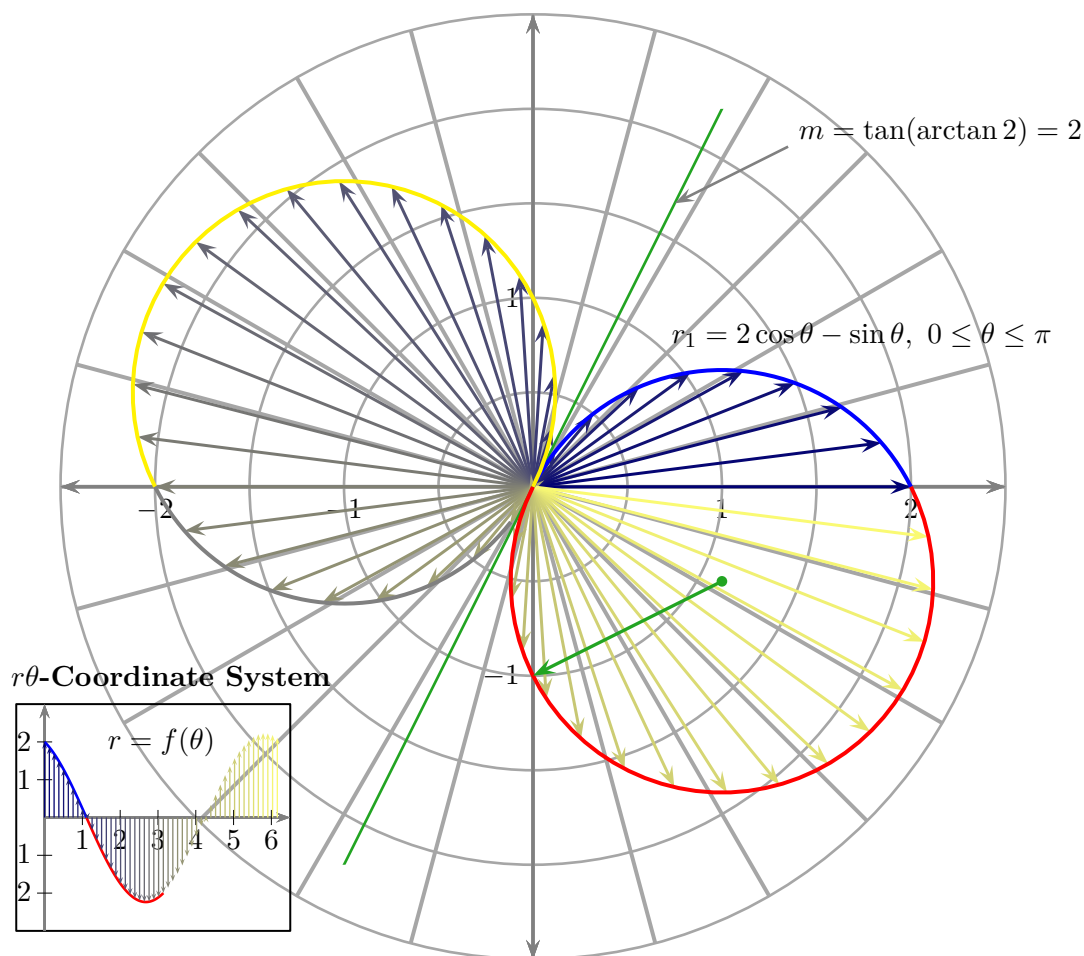
$$\begin{aligned} \Gamma(n+1) &= \int_0^{\infty} t^n e^{-t} dt \\ &= -t^n e^{-t} \Big|_0^{\infty} + n \int_0^{\infty} t^{n-1} e^{-t} dt \\ &= 0 + n\Gamma(n) \end{aligned}$$

and Euler had found his extension. That is, for each nonnegative integer n , he could now define the factorial by

$$n! = \Gamma(n+1) \tag{2}$$

Sketch the curve given by the equation below in polar coordinates.

$$r_1 = f(\theta) = 2 \cos \theta - \sin \theta, \quad 0 \leq \theta \leq \pi \quad (3)$$



This busy sketch requires some explanation. Recall that the given (polar) equation defines a circle of radius $\sqrt{5}/2$ centered at $(1, -1/2)$. The blue part of circle identifies that portion of polar equation $r_1 = 2 \cos \theta - \sin \theta, 0 \leq \theta \leq \arctan 2$, i.e., when $r_1 \geq 0$. The red part identifies the part of the curve for $\arctan 2 \leq \theta \leq \pi$, i.e., when $r_1 < 0$.

The yellow portion is actually the sketch of the curve

$$r_2 = |f(\theta)|, \quad \arctan 2 \leq \theta \leq \pi \quad (4)$$

The vectors (arrows) trace out the curve r_2 for $0 \leq \theta \leq 2\pi$. Notice that the equation in (4) yields two circles which are symmetric about the line $y = 2x$ (shown in green).