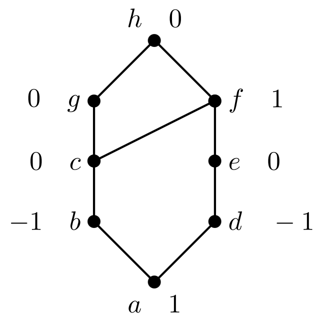

Figure 1: Poset P

1. The Hasse diagram for the poset P is shown in Figure 1. Answer the questions below.

   (a) (3 points) Compute $\mu(b, h)$.

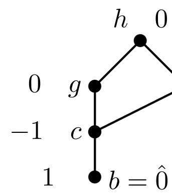

Figure 2: Poset Q

Solution:

See Figure 2.

   (b) (7 points) Find $\mu(\hat{0}, x)$ for all $x \in P$. Indicate these values in Figure 1 as we have done in class.

Solution:

See Figure 1.

2. (10 points) Let P be a locally finite poset with a minimum element $\hat{0}$ and suppose that $y \in P$ covers just one element $x \in P$. If $x \neq \hat{0}$, show that $\mu(\hat{0}, y) = 0$.

Solution:

First observe that if $x \neq \hat{0}$ we have

$$0 = \sum_{z \in [\hat{0}, x]} \mu(\hat{0}, z) \tag{1}$$

Now

$$\begin{aligned}
\mu(\hat{0}, y) &= -\sum_{\hat{0} \leq z < y} \mu(\hat{0}, z) \\
&= -\sum_{\hat{0} \leq z \leq x < y} \mu(\hat{0}, z) \quad \text{(since } y \text{ covers } x\text{)} \\
&= -\sum_{z \in [\hat{0}, x]} \mu(\hat{0}, z) \\
&= 0
\end{aligned}$$

Here the last line follows from (1).