1. (20 points) Two positive integers are called *relatively prime* if their greatest common divisor is 1. For example, 12 and 25 are relatively prime but 18 and 21 are not. Now select $n+1$ different integers from the set $[2n] = \{1, 2, 3, \ldots, 2n\}$. Show that there are always two (at least) among the selection that are relatively prime.

   Note: We often write $\gcd(m, n)$ or simply (m, n) to identify the greatest *greatest common divisor* of the integers m and n. The latter notation is used when the context is clear. So the equation $(m, n) = 1$ is another way to indicate that m and n are relatively prime.

   Solution:

   Notice that if $k \in \mathbb{P}$, then $\gcd(k, k+1) = 1$, i.e., consecutive integers are relatively prime.

   Now create n boxes $B_1, B_2, \ldots, B_n$ and assign integers $1, 2 \in B_1$, $3, 4 \in B_2$,..., $2n-1, 2n \in B_n$. Now if $n+1$ balls are distributed among the n boxes (indicating that either of the integers were chosen), then by the PHP there is at least one box with two balls. In other words, consecutive integers were chosen and we are done.