Name: _____ Student ID: _____ Section: _____

Instructions. Grading is based on method. SHOW ALL WORK.

Please submit solutions at the beginning of class on Wednesday.

1. (3 points) How many terms are in the sum below? JUSTIFY YOUR RESPONSE.

$$\sum_{n_1+n_2+\cdots+n_k=n} \binom{n}{n_1, n_2, \ldots, n_k} x_1^{n_1} x_2^{n_2} \cdots x_k^{n_k}$$

For example, $\sum_{n_1+n_2=3} \binom{3}{n_1,n_2} x_1^{n_1} x_2^{n_2} = x_1^3 + 3x_1^2x_2 + 3x_1x_2^2 + x_2^3$ is a sum of 4 terms.

2. (7 points) Give a combinatorial proof of the identity below.

$$4^k = \sum_{j=0}^{k} 3^j \binom{k}{j}$$

3. (10 points) Consider the 9-letter "word", HTHHTTHHH and answer the questions below.

   (a) How many <u>different</u> arrangements are possible?

   (b) How many different arrangements are possible if consecutive T's are forbidden? JUSTIFY YOUR RESPONSE.