Date	**Section**	**Exercises**** (QC - Quick Check and CE - Class Exercises)
08/25	1.1	CE - 1, 4, 17, 19, 38, 40
08/27	1.2	CE - 16, 18, 22, 23, 26
08/29	3.1	CE - 4, 17, 32, 37(a), 39, prove Theorem 3.5
09/03	3.2	CE - 8, 12, 18, 27, 52
09/05*	3.3	CE - 10, 25, 34, 41, 44, prove Theorem 3.21
09/08*	4.1	CE - 4, 10, 41, 46
09/10*	4.1	CE - 3, 31
09/12	4.1	CE - 21, 23, 39, 40
09/15*	4.2	QC - 1, 2; CE - 18 (give algebraic and combinatorial proofs), 19, 34
09/17*	4.3	QC - 1, 3; CE - 27
09/19*	5.1	QC - 1, 2; CE - 18, 24
09/22	5.1	QC - 3; CE - 21, 25
09/22*	5.2	QC - 1, 2; CE - 2, 19, Find formulas for $\left\{ n \atop 1 \right\}, \left\{ n \atop n-1 \right\}, \left\{ n \atop n \right\}$
09/24*	5.2	QC - 3; CE - 16, 28, 33
09/26	Bin. Inv.	1, 2, 3
09/26	Bin. Inv.	4,5

09/05 How many lottery tickets are possible in each of the modified versions of MI47 described below?

a. Each ticket has 6 numbers between 1 and 47, but now a ticket can match any number at most twice. For example, $\{1,3,3,6,42,42\}, \{2,5,10,17,31,46\},$ and $\{4,4,19,19,36,36\}$ are valid tickets, but $\{6,6,12,12,12,35\}$ is not.

b. Once again, each ticket has 6 numbers between 1 and 47, but this time a ticket can match any number as often as possible. For example, in addition to the examples in part (a), $\{4,5,21,21,21,21\}$ and $\{7,7,7,7,7,7\}$ are also a valid tickets.

09/08 Find two proofs of the identity below.

$$\frac{1}{1-x} = \sum_{n\geq 0} x^n$$

09/10 For $n \geq m$, show that

$$\sum_{k=0}^{n} \binom{n}{k} \left(\!\!\binom{k}{m}\!\!\right) (-1)^k = (-1)^n \delta_{nm} \tag{1}$$

Hint: Let $\mathcal{E}$ be the set of ways to choose an even number of candidates from $[n]$ and then allow m votes to be distributed among these candidates. In a similar manner, let $\mathcal{O}$ be the set of ways to choose an odd number of candidates. Now find a bijection between $\mathcal{E}$ and $\mathcal{O}$.

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09/15 Verify the following identities.

$$\binom{\alpha}{n} = (-1)^n \binom{n-\alpha-1}{n} \tag{2}$$

$$\sum_n \binom{n}{k} x^n = \frac{x^k}{(1-x)^{k+1}} \tag{3}$$

Hint: To prove (3), combine (2) with the General Binomial Theorem.

09/17 Let $n, k \in \mathbb{P}$. Find a combinatorial proof of the identity below.

$$\left(\!\!\binom{k}{n-k}\!\!\right) = \binom{n-1}{k-1}$$

09/19 Let $n, k \in \mathbb{N}$ with $(n,k) \neq (0,0)$. Find a combinatorial proof of the identity below.

$$\left(\!\!\binom{n}{k}\!\!\right) = \left(\!\!\binom{n}{k-1}\!\!\right) + \left(\!\!\binom{n-1}{k}\!\!\right)$$

Do not use the identity $\left(\!\binom{k}{n}\!\right) = \binom{k+n-1}{k-1}$.

09/22 Write the given set partition in block form or canonical form, as appropriate.

(a) If $\sigma = 14/238/5/67$ then $w(\sigma) =$

(b) $w(\delta) = 1123124451$ then $\delta =$

09/24 Prove each of the following identities.

(a)

$$x^n = \sum_{k=0}^{n} k! \left\{ {n \atop k} \right\} \binom{x}{k}$$

(b)

$$\sum_{k=0}^{n} (-1)^k k! \left\{ {n \atop k} \right\} = (-1)^n$$

(c)

$$\sum_{k=0}^{n} (-1)^k k! \left\{ {n \atop k} \right\} \binom{x+k-1}{k} = (-x)^n$$

09/26 (a) Find and verify a recursion formula for $k!\left\{ {n \atop k} \right\}$.

(b) Find and verify a formula for $\left\{ {n \atop 2} \right\}$. *Hint:* Use the canonical formulation of set partitions.

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