Last Time:

Given paths of almost complex structures \( J_s \), \( s \in [0, 1] \):

\[ J_s : TS^m \to TS^m, \quad J_s^2 = -1 \]

We can assign

\[ f_s : CF^0(J_0) \to CF^0(J_s) \]

\[ f_s(x) = \# \mathcal{M}(\phi) \cdot \eta \]

\# \text{ \( J_s \)-holomorphic maps}

\( \mu : \text{Sym}^k \to \mathbb{R} \) connecting \( \phi \) to \( \gamma \).

\( \mu(x, t) \)

\[ \begin{array}{c|c}
 s & \phi \\
 \hline
 0 & \phi_0 \\
 e & \phi_e \\
 1 & \phi_1 \\
 \end{array} \]

To a path of paths of a.c.s.

\[ J_{s, e} : \quad s, e \in [0, 1] \]

\( J_{s, 0} = J_s, \quad J_{s, 1} = J_s' \)

Assign

\[ H_{J_{s, e}} : CF^0(J_0) \to CF^0(J_1) \]

\[ H_{J_{s, e}}(x) = \# \mathcal{M}^e(\phi) \cdot \eta \]

\# \mathcal{M}^e(\phi) := \# \text{ pts in the moduli space}

\[ \mathcal{M}^e(\phi) = \bigcup_{e \in [0, 1]} \mathcal{M}_{J_{s, e}}(\phi) \]

where \( \phi \in \pi_2(x, y) \) has \( \mu(\phi) = -1 \).

Recall:

\[ g_1(x) - g_2(y) = \mu(\phi) - 2\eta(\phi) \]

\[ = -1 \]
Prop. \[ f_{J_3} \circ \partial_{J_3} + \partial_{J_3} \circ f_{J_3} = 0 \] (mod 2)

and \[ \partial_{J_3} \circ H_{1,2} + H_{1,2} \circ \partial_{J_3} + f_{J_3} + f_{\bar{J}_3} = 0. \]

Pf. (J₀ both cases) follows from Gramov compactness and gluing.

To complete invariance under choice of J₃, consider \( J_{1,5} + J_{6} = J_{0} \).

Prop. \[ f_{J_{1,5}} \circ J_{1,5} + f_{J_{6}} + \partial_{J_{6}} H_{5,7} + H_{5,7} \circ \partial_{J_{6}} = 0 \]

Lemma. \[ f_{J_{1,5}} \circ J_{1,5} + f_{J_{6}} = f_{J_{6}} + \partial_{J_{6}} H + H \circ \partial_{J_{6}} = 0 \]

Pf. Exercise. (Assume Gromov compactness) (Hint: \( J_{0} \subset J_{1,5} \circ J_{1,5} \circ J_{0} \))

The lemma implies the Prop: \( f_{J_{1,5}} \circ f_{J_{1,5}} + f_{J_{6} + \partial H + H \circ \partial J_{6}} = 0 \)

Lemma: \( \hat{f}_{J_{6}} \equiv Id. \)

Pf. \[ \hat{f}_{J_{6}} (\xi) = \sum_{\eta \in \Lambda} \sum_{\mu \in \mathcal{M}_{J_{6}} (\mu)} U_{\eta, \mu} (\xi) \cdot \nu_{\eta, \mu} \]

So here, \( M(\phi) \) is negative dimension unless \( R \)-action is free, i.e., for \( \phi = c \cdot w, \phi \cup \mu \).

\[ \Rightarrow \hat{f}_{J_{6}} (\xi) = 1 \cdot \xi \] if \( \phi \cup \mu \) is free.
Cor. $H_2(CF(S^2, g, h))$ remains unchanged under isotopies of $\alpha$-curves or $\beta$-curves which do not introduce new intersection pts.

Independence

1. A.C. 5. $\checkmark$ Chain maps associated to paths $\mathcal{S}_c$, inducing $\Rightarrow$ (a) $\beta$-curves
   (i) Introducing new intersection pts.
   (ii) The rest $\Rightarrow$ Follows from (1)

(b) Handle slides

(c) Stabilization $\Rightarrow$ Prove for $HF$ via gluing then for disconnected domains

Handle Slide Invariance

Idea: To a handle slide, we actually obtain a H.D. with 3 sets of curves $(\Sigma, \alpha, \beta, \gamma, \tau)$

Handle Slide Invariance

We have $\gamma_1, \ldots, \gamma_{p-1}$ an isotopy to $\beta_{g_1}, \ldots, \beta_{g_{p-1}}$.

$\gamma_g = (\beta_g \text{ skd out something else})$

Hedgehog Triple Diagram $\Rightarrow$ 3 Hedgehog Diagrams

$(\Sigma, \alpha, \beta, \tau) \Rightarrow Y$

$(\Sigma, \alpha, \gamma, \tau) \Rightarrow Y$

$(\Sigma, \beta, \delta, \tau) \Rightarrow S^1 \times S^2$

* This would not be an admissible diagram. So we need to weld $(\Sigma, \alpha, \gamma, \tau, \text{ spin C structure on } S^1 \times S^2)$. (Small admissibility perturbation)
Want a chain map between \( CF(\mathbb{Z}, \mathbb{Z}, \mathbb{Z}, \mathbb{Z}) \rightarrow CF(\mathbb{Z}, \mathbb{Z}, \mathbb{Z}, \mathbb{Z}) \).

We'll define chain maps by counting holomorphic triangles.

\[ \begin{align*}
\alpha & \rightarrow \Gamma_\alpha \\
\beta & \rightarrow \Gamma_\beta \\
\gamma & \rightarrow \Gamma_\gamma
\end{align*} \]

Want to count \( J \)-holomorphic Whitney triangles.

Prop. \( f_{\mu, \alpha} : CF^0(Y_{\mu}) \otimes CF^0(Y_{\alpha}) \rightarrow CF^0(Y_{\alpha}) \)

\[ f_{\mu, \alpha}(x \otimes y) = \sum_{r \in T_{\mu, \alpha}} \sum_{\gamma \in T_{\gamma}} \# M(\gamma) \cdot U_{\mu, \alpha}(x, y, r) \]

\( \text{Note:} \) 

\[ \begin{align*}
\phi & \rightarrow (\text{Sym } (\mathbb{Z}), T_\alpha, \ldots, T_\beta) \\
\{ \alpha, \beta, \gamma \} & \rightarrow T_\beta(\alpha, \beta, \gamma, \mu) = \{ \text{homotopy classes of} \}
\end{align*} \]

\[ \# M(\gamma) = \# \text{ of } J \text{-holomorphic squares in } \text{homotopy class of } \gamma, \]

and so forth... We can consider Whitney \( n \)-gons and \( J \)-holomorphic Whitney \( n \)-gons.
\[ P_\pi \cdot \frac{\mathbb{F}_q}{\mathbb{F}_q^*} \cong \bigoplus_{n \geq 0} \mathbb{C} \mathbb{F}_q(\mathbb{P}^1, n)^{\otimes n} \]

Hyperbolic Polynomials & Trees & $t \to \mathbb{F}_q^*$

\[ \prod_{1 \leq i < j \leq n} (\xi_i \cdots \xi_j) = \sum_{g \in \Gamma_n} \sum_{\gamma \in \Gamma_n} \# \mathcal{M}(\emptyset, \gamma) \cdot N_{g}(\gamma) \cdot \gamma \]

$\gamma \in \Gamma_n$, $\gamma(+) = 2n$

$\gamma \in \Gamma_n$, $\gamma(+) = 2n$

$\gamma \in \Gamma_n$, $\gamma(+) = 2n$

\[ \dim \left( \mathcal{M}(\emptyset, \gamma) \right) = n - 3 \quad (n > 3) \]

Moduli space of conformal $n$-gons, if $\mu(\gamma) > 3 - n$

\[ \mathcal{M}(\gamma) = \bigcup_{\gamma \in \mathcal{M}(\gamma)} \mathcal{M}(\gamma, \text{specific}) \]

\[ \delta \in \mathcal{M}(\gamma) \]

$\dim = n - 3$

is a chain map.
2. Define chain maps by counting $J$-holomorphic Whitney n-gons.

\[ \text{Prop. } \text{Face Maps: } CF(Y_{\text{rep}}) \otimes CF(Y_{\text{vir}}) \to CF(Y_{\text{vir}}) \]

\[ f_{\text{face}}(\cdot \otimes \cdot) = \sum_{\psi \in \Omega} \sum_{\gamma \in \Omega} \# M(\gamma) \cdot U^{\nu(\gamma, \psi)} \]

\[ H(\gamma) = 0 \]

Note: In general, the Morse index tells you your expected dimension with a fixed domain.

There's only one type of triangle with multiplicity.

Then we move for $n$-gons with $n \geq 3$.

\[ \text{Pf. Consider } M(\gamma), \text{ where } M(\gamma) = 1, \quad \forall \chi \in \Omega, \chi \neq \gamma. \]

If we have this type of degeneration, then we can decompose the Whitney triangle into a Whitney strip and two Whitney disks.

\[ \gamma = \phi * \gamma' \]

But then,

\[ M(\phi * \gamma') = M(\gamma) = 1 \]

\[ M(\phi) + M(\gamma') \]
Since $\mu(\varphi') < 0$ triangles have $\mathcal{M}(\varphi) = \emptyset$.

+ $\mu(\phi) \leq 0$ disks have $\mathcal{M}(\phi) = \emptyset$ (if $\phi$ is constant).

$\Rightarrow$ only cases which arise are $\mu(\varphi') = D$ and $\mu(\phi) = 1$.

Gromov compactness + Gluing $\Rightarrow \mathcal{M}(\varphi)$ is compact with $\partial \mathcal{M}(\varphi) = \text{pairs of } J$-holomorphic maps of $J_0$ from $a, \beta$.

For $i = 1, 2, \ldots, n - 1$,

and the $J$-holomorphic maps $\bar{f}_{i_{a}}$.

\[ \sum \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) + \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) + \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) \]

\[ \sum \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) + \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) = \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) + \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) \]

\[ \text{want:} \quad \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) = \bar{f}_{i_{a}}(\bar{z} \otimes \bar{t}) \]

Whitney $4$-gons:

\[ (z \in \mathbb{R}, \theta, 3) \]

\[ \text{Haps : } CF(Y_{M}) \otimes CF(Y_{P}) \otimes CF(Y_{S}) \rightarrow CF(Y_{M}) \]

\[ \text{Hom}(C_{s} \otimes C \otimes C \otimes C^{*}, \mathbb{Z}/2) \approx \text{Hom}(C_{s} \otimes C \otimes C^{*}) \]

\[ \text{Haps} \left( \bar{z} \otimes \bar{t} \right) = \sum_{m \in \mathbb{N}_{+}} \sum_{\eta \in \Pi_{m}(\bar{r}, \bar{s}, \bar{t})} \mu(\bar{d}) \cdot U_{\eta}(\bar{d}) \cdot \bar{m} \]

What type of map is this?

Not a chain map (for squares).
Consider $M(D) = \text{Hom}(\mathbb{C}^2, \mathbb{C}^2)$ with $\mu(D) = 0$.

$M(D) = \bigcup_{\text{holomorphic}} M(D, \overline{z}, \overline{i}) \rightarrow \text{Sym}^2$  

$1$-dim $\mathbb{C}$

$M(D)$ has a compactification $\overline{M(D)}$

Recall: $\overline{M}_{\overline{\mathbb{C}}}$ counted pts. in $\overline{M}(\overline{\phi}) = \bigcup_{\overline{\phi} \in \overline{\mathbb{C}}} \overline{M}_{\overline{\mathbb{C}}}(\phi)$

In this case, we expect $\overline{M(D)}$ to have $2$ types of ends

(i.e., boundary pts.) in its compactification,

namely those from Gromov compactness, and

those from the compactification of the space of holomorphic maps.

The moduli space of holomorphic maps is $1$-dim $\overline{\mathbb{C}}$, so parameterized by $\phi \in \overline{\mathbb{C}}$.

Said: "Fukaya Categories"

"Fried, "Lieslie the Theory""

Chapter 2

Claim (Weil): $\partial \overline{M(D)} = \bigcup \bigtimes \bar{i}$

$\text{Moduli space of}$

ends

$(0, \infty)$

Gromov compactness

ends

In $H_{\overline{\mathbb{C}}}$

In $d_{\overline{\mathbb{C}}}$

$\overline{\mathbb{C}}$
It must be that \( M(\tau) = M(\eta) = 0 \)

\[ M(\delta) = 1, \; M(D') = -1 \]

\[ A \in \mathcal{K} \left( \sum_{i \in \Delta} \right) \]

\[ \xi \cdot \left( \sum_{i \in \Delta} \right) \left( \sum_{i \in \Delta} \right) \]

\[ \sum_{i \in \Delta} \]

\[ \sum_{i \in \Delta} \]

\[ \sum_{i \in \Delta} \]

\[ \sum_{i \in \Delta} \]

\[ \sum_{i \in \Delta} \]

So,

\[ \mathcal{H} + \mathcal{H} \delta = \mathcal{G} \circ \mathcal{G} + \mathcal{G} \circ \mathcal{G} \]

Exercise: Think about pentagons.

\[ \mu = (3-n)+1 \]

Returning to earth,

\[ \mathcal{H} \rightarrow \left( \sum_{i \in \Delta} \right) \]

\[ \mathcal{G} \rightarrow \left( \sum_{i \in \Delta} \right) \]

\[ \mathcal{G} = \sum_{i \in \Delta} \beta_i \]
\[
\text{\textbf{Exercise! Show that (1) holds by Kunneth formula for } } \widetilde{HF} \text{ of connected sums: }
\]
\[
\tilde{\text{HF}}(S^1 \times S^2, s_0) \cong H_*(S^1)
\]
Lemma

(Am) Associativity Property satisfied

\[ f_{\alpha_1 \cdots \alpha_n} : CF(Y_{\alpha_1}) \otimes \cdots \otimes CF(Y_{\alpha_n}) \to CF(Y_{\alpha_1} \cdots \alpha_n) \]

Defined by counting \( J \)-holomorphic Whitney \((n+1) \)-gons

\[ \Psi = \pi_2 (X_{\alpha_1}, \cdots, X_{\alpha_n}, \gamma) \]

Prop. \[ f_{\alpha_\beta \alpha} \circ f_{\gamma \alpha \beta} (\Theta_{\alpha \beta} \otimes \Theta_{\gamma \alpha}) \]

\[ \cong \text{chain homotopy } \int_{\alpha \beta} \left( f_{\alpha \beta} \circ (L_{\alpha \beta} \otimes \Theta_{\alpha \beta} \otimes \Theta_{\gamma \alpha}) \right) \]

PF.

\[ \Theta_{\alpha \beta} \gamma \quad \Theta_{\gamma \alpha} \quad \Theta_{\alpha \beta} \]

\[ \beta \]

\[ \gamma \]

\[ \alpha \]

\[ H_{\alpha \beta \gamma} : CF(Y_{\alpha \beta}) \otimes CF(Y_{\gamma \alpha}) \otimes CF(Y_{\alpha \beta}) \to CF(Y_{\alpha \beta \gamma}) \]

\[ \mu = 0 \]

\[ H^{1,0} \]

\[ H^{0,1} \]

\[ J_{\text{stably imm}} \]

\[ f_{\alpha \beta \gamma} : CF(Y_{\alpha \beta}) \otimes CF(Y_{\gamma \alpha}) \to CF(Y_{\alpha \beta \gamma}) \]

\[ \tilde{f}_{\alpha \beta \gamma} : CF(Y_{\alpha \beta}) \to CF(Y_{\alpha \beta \gamma}) \]

\[ \tilde{f}_{\alpha \beta \gamma} : (X_{\alpha \beta}) \to CF(Y_{\alpha \beta \gamma}) \]

\[ X_{\alpha \beta} \otimes \Theta_{\alpha \beta} \]

\[
\text{The graph of } \mu^+ \text{'s in } S^1 \times S^2 \text{ gives a span graded generator.}
\]

\[\text{rank } t_{\mu} \text{ grading } = 1\]

\[
\begin{array}{c}
\text{not possible} \\
\text{since}
\end{array}
\]

Claim: \(f_{\text{Isom}} \) is a chain map.

Claim: \( \left( f_{\text{Isom}} \right)_* : HF(\Sigma_{\alpha}) \to HF(\Sigma_{\gamma}) \) is an isomorphism.

\[A \in CF(\Sigma_{\alpha}), \quad \exists \quad f(\alpha) \in CF(\Sigma_{\gamma}) \]
Ideally: $\mathbb{C}P(Y_{\nu_0}) \xrightarrow{i} \mathbb{C}P(Y_{\nu_{\nu}})$ to be $f$ boundary $= i$.

$\int_{\text{var}} (\nu) = \int_{\text{var}} (\nu \oplus \Theta \nu \nu)$, which canc $\nu$-holomorphic triangle.

Notice: For every $\nu_{\nu}$, $\exists$ a triangle involving $\nu_{\nu}$. Take the domain $D$ in each of these triangles, and $D$ elsewhere.

**Lemma:** \( \exists \, \nu \in \pi_2(\hat{\nu}, \Theta \nu, \hat{\nu} \text{ close}) \) (Homotopy class)

*Proof:*

\[
\begin{array}{ccc}
F & \xrightarrow{\iota} & \Sigma X \xrightarrow{\rho_1} \Sigma E \xrightarrow{\iota^*} \nu \text{ close}
\end{array}
\]
Lemma 2 \[ \mu(\mathcal{U}) = 1. \]

Pt: Apply Riemann Mapping Theorem to the \( q \)-tuple of maps involved in \( \mathcal{U} \). 

Thus, \[ \left< f \left( \mathcal{B}_{\mathcal{U}} \right), \frac{\bar{y}}{\mu(x)} \right> \neq 0 \quad (\mu \neq 0, \ldots, 2) \]

Q: How do we know there are no other homotopy classes?

A:

Exercise: 1. Define \( \Gamma_{\mathcal{U}}(\mathcal{B}, k_1, \ldots, k_r, \beta) \) in Heegard (n+1)-fold diagram.

Define \( \varepsilon(k, \ldots, k_1, \beta) \) which characterizes when

\[ \pi_3\left( \mathcal{B}, k_1, \ldots, k_r, \beta \right) \neq 0. \]

(2) When does \( \varepsilon \) live \( \mathbb{Z} \)? (See group)

(3) When \( \varepsilon = 0 \), draw the domain of a Whitney (n+1)-gon in \( \mathcal{T}_2 \). BEWARE

(4) If \( \varepsilon = 0 \), how many \( \Psi \in \mathcal{T}_2 \) are there?

(For disks, \( \mathbb{Z} \otimes \mathcal{T}_2 = \mathcal{H}_0 \left( \mathcal{T}_2 \right) \).

(5) Check that for the identity Heegard triple diagram in position \( \pi_3(\mathcal{B}, \beta, \theta) \)

With these exercises, we claim:

\[ \pi_3\left( \mathcal{B}, \theta, \mathcal{T}_2 \right) \leftrightarrow \mathbb{Z} \otimes \mathcal{T}_2 \]

\( \mathcal{T}_2 = \mathcal{T} \times \mathcal{T} \times \mathcal{T} \times \mathcal{T} \).

Any \( \mathbf{T} \) is \( \mathcal{T} \times \mathcal{T} \times \mathcal{T} \times \mathcal{T} \).

But admissibility guarantees that \( \mathcal{T} \times \mathcal{T} \times \mathcal{T} \times \mathcal{T} \) have \( \Theta \) and \( \Theta \) coefficients.

Since \( \mathcal{T} \) is so small, this ensures that the domain of \( \mathcal{T} \)

\[ \neq 0 \quad \mu(\mathcal{T}) = \emptyset. \]
Nice trick

We've shown that certain chain maps have terms which look like what we want:

+ These terms are realized by geometric objects (triangles) which are very small (in area).

Def. A filtered chain complex is a chain complex $(C_\bullet, d)$ plus a map $\mathcal{F}: C_\bullet \to \mathbb{R}$ such that $\mathcal{F}(d(x)) > \mathcal{F}(x)$.

Def. A filtered chain map is a chain map $g$ between filtered complexes $\mathcal{F}$, $\mathcal{F}'$ such that $\mathcal{F}'(g(x)) \leq \mathcal{F}(x)$.

Lemma. Suppose $g: (C, d, \mathcal{F}) \to (C', d', \mathcal{F}')$ is a filtered chain map, and $g = \text{isomorphism} + \text{lower order terms}$

\[ g(x) = i(x) + lb(x) \]

with $\mathcal{F}'(lb(x)) < \mathcal{F}'(i(x))$ for.

Then $g_\# : H_\ast(C) \to H_\ast(C')$ is an isomorphism.

Proof. Choose filtered bases for $C$ and $C'$.

Write the basis:

\[
\mathcal{F} = \begin{bmatrix}
\delta & 0 \\
* & \\
\end{bmatrix}
\]

Then, a filtered change of basis doesn't change $g_\#$.

Therefore, we can perform column operations in change of matrix to $\text{Id}$. 0

Conclusion of Proof:

Defining $f$ induces an isomorphism (on homology).

Pick filtrations on $\mathcal{F}(Y_\bullet, \rho)$, $\mathcal{F}(Y_\bullet, \tau)$ by

$\mathcal{F}(X_{\text{orbing}}) = 0$, $\mathcal{F}(X_{\text{else}}) = -\text{Sign} \text{Area}(D(\delta))$

or

$\mathcal{F}(X_{\text{orbing}}, X_{\text{else}})$.

$\mathcal{F}(Y_\bullet, \rho)$ is filtered by \( M(\delta) \neq \emptyset \Rightarrow \text{Area}(D(\delta)) \leq 0 \).
They show the triangle map as intended, the small triangles preserved the filthness, and every other triangle (including one near) decreases the filthness.
Last time: We showed that for \( f: \hat{\mathcal{L}}' \to \mathbb{M}' \),

\[
(f_{\mathcal{L}'})_{\mathcal{L}}: \text{HF}(X_{\mathcal{L}}) \cong \text{HF}(X_{\mathcal{L}'})
\]

\[
\sigma_{\mathcal{L}}
\]

\[
\begin{pmatrix}
1 & 0 \\
* & 1 \\
* & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 1 \\
0 & \text{Chern indices}
\end{pmatrix}
\]

Note: \( r_k(\text{CF}(X_{\mathcal{L}})) = n \)

\[ r_k(\text{CF}(X_{\mathcal{L}'}) = n + k, \quad \mathbb{Z}_{20} \]

Note: From last time, we showed that the chain map looks like

\[
\begin{pmatrix}
1 & 0 \\
* & 1 \\
* & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 1 \\
0 & \text{Chern indices}
\end{pmatrix}
\]

So, \( \text{HF is independent of HD up to isotopy of curves.} \)

It remains to show \( \text{HF is independent up to homotopy of Ronalds.} \)
\[ \Psi_{\gamma} (f_{\alpha \gamma} (\xi \Theta_{\alpha \gamma} \otimes \Theta_{\gamma}^{\tau} \Theta_{\tau}^{\gamma})) \]

This picture of the handle slide doesn't come without work, even if it is a curve.

The next step is to show:

\[ \Psi_{\gamma} (\Theta_{\alpha \gamma} \otimes \Theta_{\gamma}^{\tau} \Theta_{\tau}^{\gamma}) = \Theta_{\gamma} \]

Finally, \[ \Psi_{\gamma} (\xi \Theta_{\gamma}) = \psi \text{ for } (\xi) \]

is an isomorphism on homology (as we showed last time).

So, \[ \Psi_{\gamma} (f_{\alpha \gamma} (\xi \Theta_{\alpha \gamma} \otimes \Theta_{\gamma}^{\tau} \Theta_{\tau}^{\gamma})) \]

is an isomorphism on homology.

\[ \Rightarrow \quad f_{\gamma \delta} (\xi \Theta_{\gamma}) = \psi \text{ for } (\xi) \]

is injective on the level of homology.

To show surjectivity, we will "cyclically permute the argument."

Let \( y \) be parallel (but admissible) to \( x \).

Let \[ (2 \in \delta, \gamma) \]

And \[ (\xi \Theta_{\gamma}) = \Theta_{\gamma} \]

And \[ f_{\gamma \delta} (\xi \Theta_{\gamma}) = \psi \text{ for } (\xi) \]

is an isomorphism on homology.

\[ \Rightarrow \quad f_{\gamma \delta} \text{ is surjective} \] (remembering that \( \delta \) is metric to \( \beta \)).
Compute \[ \text{CF}(Y_{st}), \mathcal{J} \] and \[ H_a(T^d) \] of \( \text{CF}(Y_{pr}) \).

\[ \text{Compute } \int_{\operatorname{Ext}} (\Theta_{pr} \otimes \Theta_{x^d}) \]

\text{Claim:} The relative grading for this diagram is the same as the previous.

\[ q^r(\nu^+ \nu_2^+) \nu_1^+ \nu_2^-) = \text{M}(\phi) - 2 \nu_c(\phi) - 1 \]

\text{Clock - } \theta \text{ All other domains differ from these two and } \phi. \text{ By periodicity, } \nu \text{ and } \bar{\nu} \text{ coefficients,}

\text{there are only 3 domains to worry about, one of which is covered by the R.M.T.}
Claim: \( \# \mathcal{M}(\phi_2) + \# \mathcal{M}(\phi_3) = \pm 1 \).

For each domain

If \( \text{cut} \) length \( \beta = 0 \), modulus is a circle, which has no holomorphic rep.

Any cut for \( \beta \) near zero is greater than exterior.

Thus, \( HF \) is an invariant of a closed, oriented 3-manifold.

In fact, our proof shows that \( HF^0(\mathcal{Y}, \phi) \) are invariants when \( \phi \) is a relative \( \mathbb{Z} \)-grading.

\[
\text{div}(c_1(s)) = \gcd (\langle c_1(s), \omega \rangle, \alpha) = 0 \\
\alpha \in H_2(\mathcal{Y}).
\]

Also, the spin \( \text{spin}^c(\mathcal{Y}) \) splitting is an invariant.

\[
\text{CF}(\mathcal{Y}_{\text{top}}, s) \xrightarrow{\text{max}} \text{CF}(\mathcal{Y}_{\text{max}}, s) \\
\text{Se}(\mathcal{Y}) = s \quad \rightarrow \quad \text{Se}^{-1}(\text{fiber}(s)) = s.
\]

Invariance proof considered maps induced by \( Y \times I = W \).
Considering $BD$'s and sets of base points $(\xi, z, \beta, \tau, \omega, \ldots)$ gives rise to knot + link invariants, invariants of 3-manifolds.

Using the $A_m$-associativity of $J$-holomorphic pentagon, and a triangle count, we can obtain e.g. relating $HF$ of 3-manifolds differing by surgery along $K$.

$M, \quad \partial M = \Sigma$

Thus $(\alpha, \beta, \gamma) \in \Sigma \setminus \Sigma^+$. S.t. $\alpha \cdot \alpha = 1, \beta \cdot \beta = 1, \beta \cdot \gamma = 1$.

Then $(M, \alpha) \to M(\alpha) = M \cup S^1 \times D^2$

$\partial D^2 = \infty$

$\to \hat{HF}^+(M(\alpha)) \to \hat{HF}^+(M(\beta)) \to \hat{HF}^+(M(\gamma)) \to \ldots$