
All answers must be justified appropriately.

From TREIL do the following exercises.

(Note that when we refer to Exercise a.b.c, we mean Exercise b.c from Chapter a.)

page 46: 2.2.1(a),(b),(c),(d): **Make sure you solve the systems!**

page 46: 2.2.2

page 51: 2.3.1, 2.3.3, 2.3.6

page 55: 2.5.1 (a)-(g) (Here “appropriate justification” is brief, just one or two sentences.)

1. Find the inverse C^{-1} of the matrix

$$C = \begin{pmatrix} 1 & -1 & -2 \\ 1 & 1 & -2 \\ 1 & 1 & 4 \end{pmatrix} \in \text{Mat}_{3 \times 3}(\mathbb{Q})$$

and write C^{-1} as a product of elementary matrices

$$E = E_N E_{N-1} \cdots E_2 E_1.$$

(Make sure you show the matrices E and E_i , $i = 1, \dots, N$.)

2. Using elementary row operations, write the matrix A given by

$$A = \begin{pmatrix} 1 & -2 & 0 & 2 & -3 \\ 2 & -4 & 2 & 0 & 8 \\ 1 & -2 & 3 & -3 & 16 \end{pmatrix} \in \text{Mat}_{3 \times 5}(\mathbb{Q})$$

as EA where EA is in reduced row echelon form (RREF) and E is a product of elementary matrices,

$$E = E_N E_{N-1} \cdots E_2 E_1.$$

(Make sure you show the matrices E and E_i , $i = 1, \dots, N$.)

3. Let W be a subspace of the finite dimensional vector space V over \mathbb{F} .

- (i) Prove that $\dim_{\mathbb{F}}(W) \leq \dim_{\mathbb{F}}(V)$.
- (ii) Prove that $\dim_{\mathbb{F}}(W) = \dim_{\mathbb{F}}(V)$ if and only if $W = V$.

HINT: We have a result (Corollary 1.7(c) of the Supplement and Proposition 2.5.4 of TREIL) that says, “In a finite dimensional vector space, every linearly independent system is contained in a basis.”