6.4. The Nonlinear Pendulum

Section Objective(s):
- Nonlinear Pendulum as a First Order System.
- Critical Points and Linearization.
- Phase Portrait with and without Friction.

Remarks:

- We study in detail the _____ equations.

- We write it as a _____ system.

- We find the _____ and the _____ at these _____.

- We sketch a _____ of the solutions in the case without and with small _____.

6.4.1. **Nonlinear Pendulum as a First Order System.**

Definition 1. A *pendulum* is a small ball _____ attached

to the end of a rigid rod _____ , the latter can swing from its

other end in a medium _____ .

EXAMPLE 1: Show that the equation of motion of a pendulum are given by

where _____ is the angle as function of time of the pendulum with the downward

vertical, positive counter-clockwise.

SOLUTION:

◁
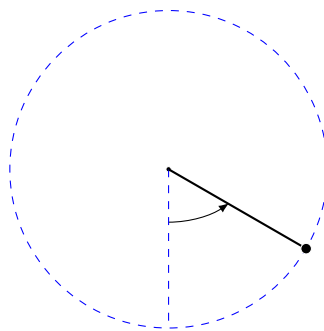
6.4.2. **Critical Points and Linearization.**

Remark: We consider the particular case _____ and _____,

We also consider the case of no friction, _____, or small friction, _____.

EXAMPLE 2: Find the critical points of the nonlinear pendulum.

SOLUTION:

EXAMPLE 3: Find the linearization of the pendulum equations at the critical points.

SOLUTION:

◁

6.4.3. **Phase Portrait with No Friction.**

Remark: Use the interactive graph below to help you find the phase portrait of the nonlinear pendulum.

  Interactive Graph Link.

Example 4: Sketch a phase portrait of the nonlinear pendulum with no friction.

Solution:

6.4.4. **Phase Portrait with Small Friction.**

Example 5: Sketch a phase portrait of the nonlinear pendulum with small friction.

Solution: