## 5.5. Diagonalizable Matrices

**Section Objective(s):**
- Diagonal Matrices.
- Diagonalizable Matrix.

**Remarks:**

- ____________________ matrices are simple to work with, but they ________________________________________ in physical applications.
- __________________ matrices are difficult to work with, since the matrix product is __________________________________________________________.
- ____________________________ matrices are an intermediate case:
  - They are ____________________________ to often appear in physical applications.
  - The are ___________________________ to work with.
  - Functions of ___________________________ matrices are _______________ to compute.

### 5.5.1. Diagonal Matrices.

**Definition 1.** An $n \times n$ matrix $A$ is ____________________ iff

**Remarks:**

- Notation:
- Matrix operations are ________________ with diagonal matrices.

EXAMPLE 1: Given $A = \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix}$, compute $A^2$, $A^3$, and $A^n$ for a general natural number $n$.

SOLUTION:

$\triangleleft$

**Remarks:** Consider a diagonal matrix $D = \operatorname{diag}\big[a_{11}, \cdots, a_{nn}\big]$:

- Then $D^n =$ ______________________________________________________.
- The eigenvalues of a $D$ are ________________________________________.
- The corresponding eigenvectors are

### 5.5.2. Diagonalizable Matrices.

**Remarks:**

- Diagonal matrices __________________ appear often in physical applications.
- But __________________________ matrices are very common in physical aplications.

**Definition 2.** A square matrix $A$ is ________________________________ iff there exists an invertible matrix $P$ and a diagonal matrix $D$ such that

**Remark:** __________________________ is equivalent to __________________________.

Example 2: Show that the matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ is diagonalizable with $P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

Solution:

$\lhd$

EXAMPLE 3: If $A$ is a $2 \times 2$ with eigenpairs $\lambda_1$, $\boldsymbol{v}_1$ and $\lambda_2$, $\boldsymbol{v}_2$, then show that

$$AP = PD, \qquad \text{where} \qquad P = \big[\boldsymbol{v}_1, \boldsymbol{v}_2\big], \qquad D = \operatorname{diag}\big[\lambda_1, \lambda_2\big].$$

SOLUTION:

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**Remark:** The next result says that this result and its converse are true for $n \times n$ matrices.

**Theorem 1.** An $n \times n$ matrix $A$ is ________________________________ iff $A$ has ________________________________ linearly independent.

Furthermore, if $\lambda_i, \boldsymbol{v}_i$, for $i = 1, \cdots, n$, are eigenpairs of $A$, then $A = PDP^{-1}$, where

Example 4: Is the matrix $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. diagonalizable?

Solution:

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**Remark:** Matrix $P$ is ____________________, since the eigenvectors are ____________________.

EXAMPLE 5: Is the matrix $B = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalizable?

SOLUTION:

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