5.4. Eigenvalues and Eigenvectors.

Section Objective(s):

- Eigenvalues and Eigenvectors.
- Computing Eigenpairs.

Remarks:

- A matrix acting on a vector usually _____ the direction of the vector.
- An eigenvector of a matrix A determines a particular _____ in space that is _____ under the action of A.
- The eigenvectors of the _____ of a linear _____ system will play an important role to find solutions to the system.

5.4.1. **Eigenvalues and Eigenvectors.**

Definition 1. A number λ and a nonzero n-vector $\boldsymbol{v}$ are an _____ and _____ also called _____, of a square matrix A iff they satisfy the equation

Remark: The length of an eigenvector is _____, because if $\boldsymbol{v}$ is an eigenvector, so is _____.

EXAMPLE 1: Verify that the pair λ_1, $\boldsymbol{v}_1$ and the pair λ_2, $\boldsymbol{v}_2$ are eigenpairs of matrix A,

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \qquad \begin{cases} \lambda_1 = -1 & \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \\ \lambda_2 = -2 & \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}. \end{cases}$$

SOLUTION:

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Interactive Graph: A Geometrical Meaning of Eigenpairs.

5.4.2. **Computing Eigenpairs.**

Remarks:

- The eigenpairs equation can be written as

- This equation says that the matrix _____ is _____.
- There is a way to _____ whether this matrix is _____,

Theorem 1. (Eigenvalues-Eigenvectors)

(a) All the eigenvalues λ of an $n \times n$ matrix A are the solutions of

(b) Given an eigenvalue λ of an $n \times n$ matrix A, the corresponding eigenvectors $\boldsymbol{v}$ are the nonzero solutions to the homogeneous linear system

Remarks:

- We look for _____ such that the matrix _____ is _____
- Given an $n \times n$ matrix A, the function _____ is a _____.
- This _____ is called the _____ of matrix A.

EXAMPLE 2: Find the eigenvalues λ and eigenvectors $\boldsymbol{v}$ of the matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$.

SOLUTION:

EXAMPLE 3: Find the eigenvalues λ and eigenvectors $\boldsymbol{v}$ of the matrix $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$.

SOLUTION:

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