

5.1. SYSTEMS OF ALGEBRAIC LINEAR EQUATIONS

Section Objective(s):

- The Row Picture.
- The Column Picture.
- The Matrix Picture.

Remarks:

- Before trying to solve systems of _____ we need to know basic concepts of _____.
- Solving linear algebraic equations by _____ is called the _____. (One equation at a time.)
- Linear algebraic equation can be thought as _____. This is the _____.
- The concept of _____ comes from the _____.
- Linear algebraic equation can be thought as _____. This is the _____.
- From the _____ we get the idea that matrices are _____.

5.1.1. The Row Picture.

Remark: The field of _____ started when people tried to solve systems of _____.

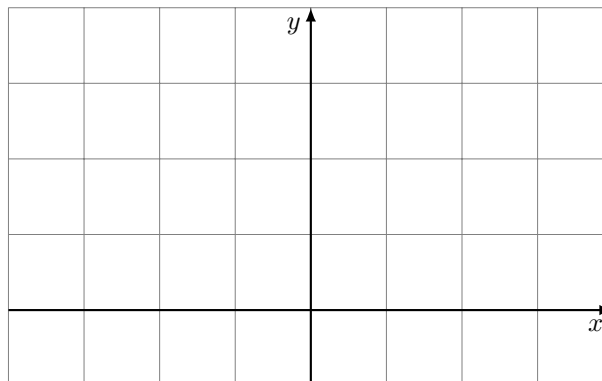
EXAMPLE 1: Find all solutions (x, y) of the 2×2 linear system

$$2x - y = 0$$

$$-x + 2y = 3.$$

Provide both a geometrical and an analytical solution.

SOLUTION:



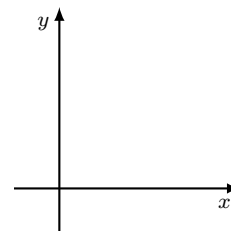
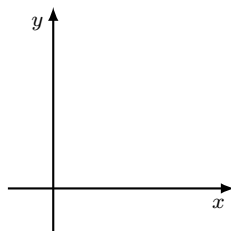
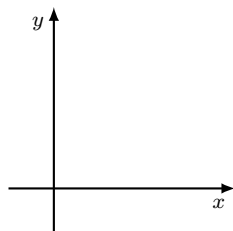
Theorem 1. Given a 2×2 linear system, only one of the following statements holds:

(i) _____

(ii) _____

(iii) _____

Proof:



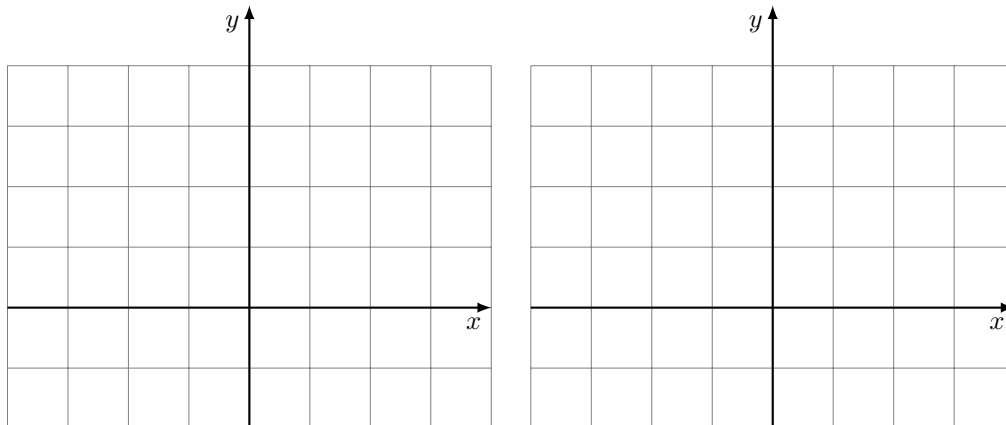
5.1.2. The Column Picture.

Remark: The concept of a _____ of vectors, and the idea of _____ come from the _____.

EXAMPLE 2: Write the system in Example 1 as a linear combination of column vectors,

$$\begin{aligned}2x - y &= 0 \\ -x + 2y &= 3.\end{aligned}$$

SOLUTION:



Remark: The example above is the motivation for the following definition.

Definition 1. The _____ of the n -vectors

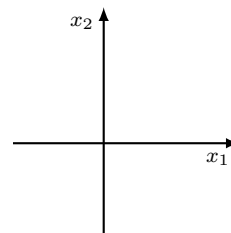
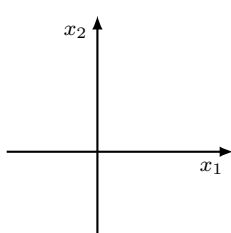
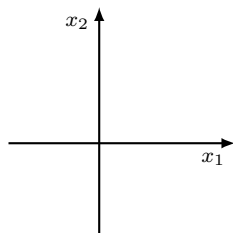
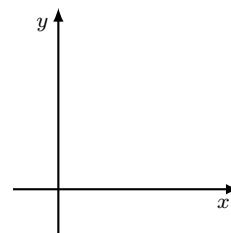
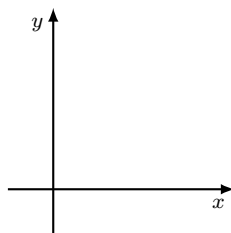
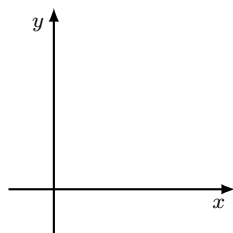
with the real numbers a and b , is defined as

Remark: Theorem 1 can also be proven using linear combination of column vectors.

Theorem 1. Given a 2×2 linear system, only one of the following statements holds:

- (i) _____
- (ii) _____
- (iii) _____

Proof:



5.1.3. The Matrix Picture.

Remark: The concept that a matrix is _____
comes from the _____

EXAMPLE 3: Write the system in Example 1 as a matrix acting on a column vector,

$$\begin{aligned} 2x - y &= 0 \\ -x + 2y &= 3. \end{aligned}$$

SOLUTION:

◁

Remark: The example above motivates the following definition.

Definition 2. An _____ is an array of numbers

m rows,
,
 n columns,

where $a_{ij} \in \mathbb{R}$ or \mathbb{C} , for $i = 1, \dots, m, j = 1, \dots, n$. A _____
is an _____ matrix, and the _____ coefficients in a square
matrix are _____.

Remark: A matrix is a _____ that acts on _____ and
the result is _____

