4.1. Modeling with First Order Systems

Section Objective(s):

- Interacting Species.
- Predator-Prey.
- Spring-Mass as a First Order System.
- Equilibrium Solutions.

Remarks:

- We have studied how to solve _____ equations

- We have also student how to solve _____ equations

- There are more complex physical system that _____ be decribed with the equations above.

- Today we see two of such systems: _____ and _____ systems.

- Then we see that _____ equations can be written as _____ systems.

4.1.1. **Review: Interacting Species.**

Example 1: Construct a differential equation that describes the population of rabbits and sheep coexisting in an environment with finite resources.

Solution:

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Definition 1. The ***interacting species system*** for the functions x and y, which depend on the independent variable t, is

where the constants r_x, r_y and x_c, x_c are positive and α, β are real numbers. Furthermore, we have the following particular cases:

- The species _____ iff $\alpha < 0 \qquad \beta < 0$ _____.
- The species _____ iff $\alpha > 0 \qquad \beta > 0$ _____.
- The species y _____ with x when _____,

  and x _____ with y when _____.

Example 2: The interaction of of rabbits and elephants is given by

which variable represents the elephants? What is the growth rate and carrying capacity of the elephants and of the rabbits?

Solution:

◁

4.1.2. **Predator-Prey System.**

Example 3: Construct a differential equation that describes the population of rabbits and foxes coexisting in an environment with unlimited resources for the rabbits.

Solution:

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Definition 2. The ***predator-prey system*** for the predator function x and the prey function y, which depend on the independent variable t, is

where the coefficients a_x, b_x, a_y, and b_y are nonnegative.

Remark:

- A predator is called _____ if they seldom catch prey but can live for a long time on a single prey, for example boa constrictors.
- A predator is called _____ if they catch prey very often and they can live for only a short time on a single prey, for example bobcats.

Example 4: Identify which of the systems below corresponds to a lethargic predator and which one to an active predator.

$$x' = 0.3\,x - 0.1\,xy, \qquad \tilde{x}' = 0.3\,\tilde{x} - 3\,\tilde{x}\tilde{y}$$
$$y' = -0.1\,y + 2\,xy, \qquad \tilde{y}' = -2\,\tilde{y} + 0.1\,\tilde{x}\tilde{y}.$$

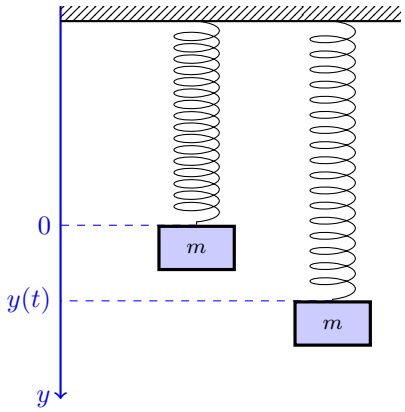
Solution:

◁

4.1.3. **Spring-Mass as a First Order System.**

Example 5. (Mass-Spring System): Consider an object of mass m mass hanging at the bottom of a spring with spring constant k, and moving in a fluid with damping constant d. Assume that there is an external force f, which depends on t, acting on the object.

If $y(t)$ is the displacement from the equilibrium position at the time t, positive downwards, the equation of motion for the variable y is



Write the differential equation above as a first order system.

Solution:

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4.1.4. **Equilibrium Solutions.**

Remark: Equilibrium solutions are defined for autonomous ssystems

Definition 3. (Equilibrium Solutions) The _____ of the autonomous system

are solutions of the form _____, for all t,

which satisfy _____

The point (x_{10}, x_{20}) in the x_1x_2 plane is called an _____ of the system.

Example 6: Find the equilibrium solutions of the following competing species system.

$$R' = 3R\,(1 - S - R)$$
$$S' = 2S\,(2 - S - 3R)\,.$$

Solution:

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