3.3. Discontinuous Sources

Section Objective(s):
- Overview: Step Functions.
- Laplace Transform of Steps.
- Translation Properties of the LT.

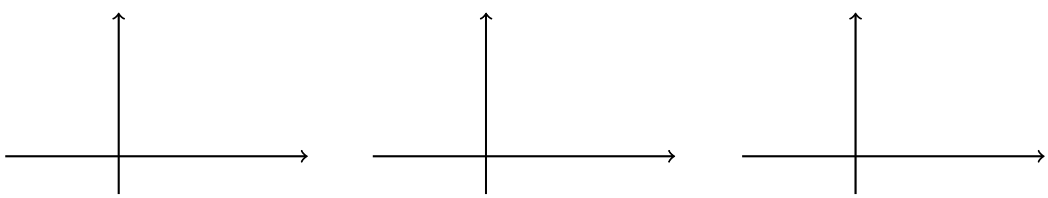
3.3.1. Overview: Step Functions.

Definition 1. The ***step function*** at $t = 0$ is

$$u(t) = \begin{cases} \underline{\hspace{2cm}} & t < 0, \\ \underline{\hspace{2cm}} & t \geqslant 0. \end{cases}$$

Example 1: Graph the step u, $u_c(t) = u(t - c)$, and $u_{-c}(t) = u(t + c)$, for $c > 0$.

Solution:



$\lhd$

Example 2: Graph the bump function $b(t) = u(t - a) - u(t - b)$, for $a < b$.

Solution:

$\lhd$

3.3.2. **Translation Identities.**

Theorem 2. (Translation Identities) If $\mathcal{L}[f(t)](s)$ exists for $s > a$, then

$$\mathcal{L}[u(t-c)f(t-c)] = \underline{\hspace{6cm}}, \qquad s > a, \qquad c \geqslant 0 \tag{3.3.1}$$

$$\mathcal{L}[e^{ct}f(t)] = \underline{\hspace{6cm}}, \qquad s > a + c, \qquad c \in \mathbb{R}. \tag{3.3.2}$$

EXAMPLE 3: Take $f(t) = \cos(2t)$ and write the equations given the Theorem above.

SOLUTION:

$\lhd$

EXAMPLE 4: Take $f(t) = 1$ and write the equations given the Theorem above.

SOLUTION:

$\lhd$

Example 5: Find the function f such that $\mathcal{L}[f(t)] = \dfrac{e^{-4s}}{s^2+5}$.

Solution:

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Example 6: Find the function $f(t)$ such that $\mathcal{L}[f(t)] = \dfrac{(s-1)}{(s-2)^2+3}$.

Solution:

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3.3.3. **Solving Differential Equations.**

EXAMPLE 7: Use the LT to find the solution to the initial IVP

$$y'' + y' + \frac{5}{4}\, y = b(t), \qquad y(0) = 0, \qquad y'(0) = 0, \qquad b(t) = \begin{cases} 1 & 0 \leqslant t < \pi \\ 0 & t \geqslant \pi. \end{cases} \tag{3.3.3}$$

SOLUTION:

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