3.2. The Initial Value Problem

Section Objective(s):

- Homogeneous IVP.
- Non-Homogeneous IVP.
- Higher Order IVP.

When can we apply the Laplace Transform Method?

(a) The ODEs have to be _____.

(b) The sources can be _____ or _____.

The big picture approach in using the LT to solve ODEs:

The One-to-One Property:

Theorem 1. (Injectivity of the Laplace Transform) If f, g are continuous on $[0, \infty)$ and bounded by an exponential, then

Remark: We use one-to-one property when we solve differential equations.

- If we LT a _____ equation for y and _____ for $\mathcal{L}[y]$, we get

- We find $h(t)$ such that _____, so we get

3.2.1. **Homogeneous IVP.**

Example 1: Use the Laplace transform to find the solution y to the initial value problem

$$y'' - y' - 2y = 0, \qquad y(0) = 1, \qquad y'(0) = 0.$$

Solution:

3.2.2. **Non-Homogeneous IVP.**

Example 2: Use the Laplace transform to find the solution y to the initial value problem

$$y'' - 4y' + 4y = 3\,e^t, \qquad y(0) = 0, \qquad y'(0) = 0.$$

Solution:

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3.2.3. **Higher Order IVP.**

Example 3: Use the Laplace transform to find the solution y to the initial value problem

$$y^{(4)} - 4y = 0, \qquad \begin{aligned} y(0) = 1, \quad & y'(0) = 0, \\ y''(0) = -2, \quad & y'''(0) = 0. \end{aligned}$$

Solution:

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