EXAMPLE 3. (EXTENDABILITY OF SOLUTIONS TO LINEAR EQUATIONS): In the initial value problems below find the **maximum domain** where the solution is **certain** to exist.

(1) $t(t-5)\,y' = y, \qquad y(-1) = 4$

(2) $(t^2-4)\,y' - 5\ln(t)\,y = 3t, \qquad y(1) = 2$

SOLUTION:

(1) The eq. is $y' = \dfrac{1}{t(t-5)}\,y \Rightarrow a(t) = \dfrac{1}{t(t-5)}$, $b(t) = 0$

The eq. is defined on $\mathbb{R} - \{0, 5\}$

That is: on $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$

The solutions y are defined on

$(-\infty, 0)$ or $(0, 5)$ or $(5, \infty)$.

The I.C. is $y(-1) = 4 \Rightarrow t_0 = -1 \in (-\infty, 0)$.

So the sol. y of the IVP is defined on

$$\boxed{D = (-\infty, 0)}$$

$\triangleleft$

$\downarrow$

(2) The eq. is

$$y' = \frac{5\ln(t)}{(t^2-4)}\,y + \frac{3t}{(t^2-4)} \Rightarrow a(t) = \frac{5\ln(t)}{(t^2-4)}$$

$$b(t) = \frac{3t}{(t^2-4)}$$

The eq. is defined on $\mathbb{R} - \{-2, 2\} \cap \mathbb{R} - (-\infty, 0)$

That is: on $(0,2) \cup (2,\infty)$.

The solutions y are defined on

$(0,2)$ or $(2,\infty)$.

The I.C. is $y(1) = 2 \Rightarrow t_0 = 1$

So, the solution y of the IVP is defined on

$$\boxed{D = (0,2)}$$