1.5. Linear Equations

Section Objective(s):

- Constant Coefficient Equations.
- Variable Coefficient Equations.
- The Integrating Factor Method.

Remarks:

- The study equations of the form _____.
- Constant coefficients _____ equations _____.
- We review _____ these equations.
- Variable coefficients _____ equations _____.
- And integrating on both sides of the equation _____.
- _____ is needed to solve _____ equations.
- _____ is to transform the linear equation into _____.
- This is what _____.

1.5.1. **Linear Constant Coefficient Equations.**

Definition 1. A ***linear differential equation*** on the function y is

The equation has _____ coefficients if both a and b are constants, otherwise the equation has _____ coefficients.

Example 1. (Constant Coefficients): Solve linear constant coefficients equations using that they are _____.

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Example 2. (Variable Coefficients with $b = 0$): Solve linear variable coefficients equations, _____, using that they are _____.

Solution:

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Example 3.: Find all solutions of The solutions of $y' = 2t\,y$.

Solution:

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Remark: The case b/a non-constant _____ be solved with this idea.

1.5.2. **Variable Coefficient Equations.**

Theorem 1. (Variable Coefficients) If the functions a, b are continuous, then

(1.5.1)

has infinitely many solutions given by

(1.5.2)

where _____ and $c \in \mathbb{R}$.

Remarks:

(a) The expression in Eq. (1.5.2) is called the _____.

(b) We solve these equations using the _____.

(c) The function _____ is the _____.

EXAMPLE 4. (INTEGRATING FACTOR METHOD): Find all the solutions of the equation

$$ty' = -2y + 4t^2, \quad \text{with} \quad t > 0.$$

SOLUTION:

Example 5. (Initial Value Problem): Find the solution to the initial value problem

$$ty' + 2y = 4t^2, \qquad t > 0, \qquad y(1) = 2.$$

Solution:

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