

EXAMPLE 3: Find all solutions of $ty' = -2y + 4t^2$, with $t > 0$.

SOLUTION:

$$y' = -\frac{2}{t} y + 4t \quad (\text{divide by } t)$$

$$y' + \frac{2}{t} y = 4t$$

$$\mu y' + \frac{2}{t} \mu y = \mu 4t$$

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$$t^2 y' + \frac{2}{t} t^2 y = t^2 4t$$

$$t^2 y' + 2t y = 4t^3$$

$$(t^2 y)' = (t^4)'$$

$$(t^2 y - t^4)' = 0$$

(total derivative)

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$$\Rightarrow \mu' = \frac{2}{t} \mu$$

$$\frac{\mu'}{\mu} = \frac{2}{t}$$

$$\begin{aligned} \ln |\mu| &= 2 \ln |t| + c_0 \\ &= \ln(t^2) + c_0 \end{aligned}$$

$$\mu = (\pm e^{c_0}) t^2$$

choose $c_0 = 0$

$$\mu = t^2$$

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$$t^2 y - t^4 = C$$

implicit sol.

$$t^2 y = C + t^4$$

$$y = \frac{C}{t^2} + t^2$$

explicit sol.

EXAMPLE 4: Find the solution to the initial value problem

$$ty' + 2y = 4t^2, \quad t > 0, \quad y(1) = 2.$$

SOLUTION:

We know : $y = \frac{c}{t^2} + t^2$

I.C. $2 = y(1) = c + 1 \Rightarrow \boxed{c=1}$

$$\boxed{y = \frac{1}{t^2} + t^2}$$