## 1.3. Qualitative Analysis

**Section Objective(s):**

- The Existence of Solutions Theorem.
- Direction Fields.
- Autonomous Equations.

**Remarks:**

- If the equation is ______________________, then _____________ solutions.
- However, there is ________________________________ for the solutions of ________ differential equations.
- The ____________________________ we know are _____________________ to write their solutions.
- Simple functions are ______________________________________________ ____________________________________________________________
- There are more __________________ than _______________________ needed to write their solutions.
- It is _______________ to study _________________________________ to describe solutions to differential equations.
- We get information about the _________________ of differential equations ___________________________ the equation.
  (a) ________________________________________, works with ________ equations.
  (b) ___________________________________________________, works with __________________ equations.

### 1.3.1. **The Existence of Solutions Theorem.**

**Theorem 1.3.1.** (Picard-Lindelöf) Consider the initial value problem

If the function $f$ and its partial derivative $\partial_y f$ are continuous on some rectangle on the $ty$-plane containing the point $(t_0, y_0)$ in its interior,

then ______________________________ of the initial value problem above on an open interval $I$ containing the point $t_0$.

**Remarks:**

(1) An ______________________________ means to find a solution to ____________ a differential equation and an initial condition.

(2) There is ________________ for the solution in this Theorem.

(3) Results with ________________ are still ____________

Example 1.3.1: Determine whether the functions $y_1$ and $y_2$ given by their graphs in Fig. 1 can be solutions of the same differential equation satisfying the hypotheses in the Picard-Lindelöf Theorem.

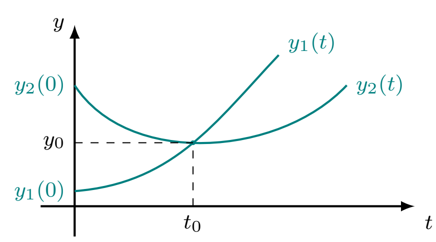


Figure 1. The graph of two functions.

Solution:

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### 1.3.2. Direction Fields.

**Remark:** We interpret $f(t, y)$ at each point $(t, y)$ on the $ty$-plane as

______________________________________________________ .

**Definition 1.6.3.** The ***direction field*** of the differential equation

is the graph on the ____________________ of $f(t, y)$

______________________________________________________.

Example 1.6.11: Find the direction field of the equation $y' = \sin(y)$, and sketch a few solutions to the differential equation for different initial conditions.

Solution:

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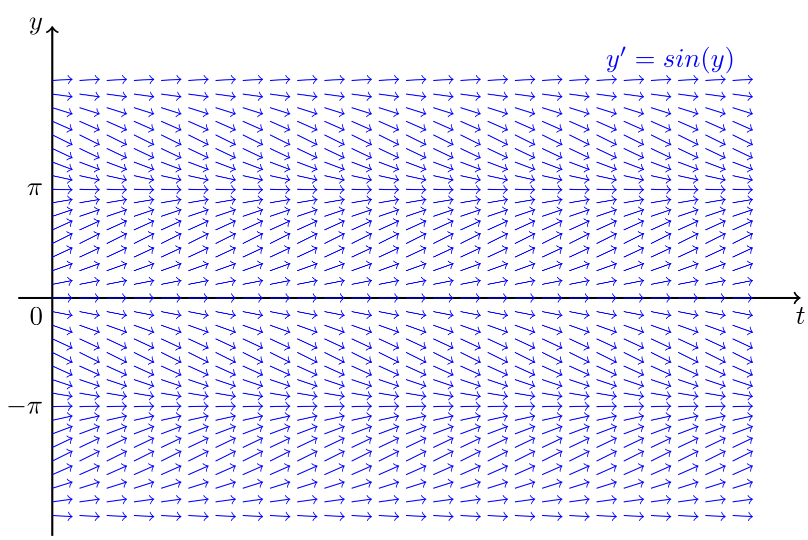


Figure 2. Direction field for the equation $y' = \sin(y)$.

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### 1.3.3. **Autonomous Equations.**

**Definition 6.1.1.** A first order equation is ________________________ iff

________________________,

where $y' = \dfrac{dy}{dt}$, and the function $f$ ________________ depend explicitly on $t$.

**Remark:** An important example of an autonomous equation is

________________________.

**Remark:** The ________________________ can be solved exactly.

Example 6.1.7: Sketch a qualitative graph of solutions of

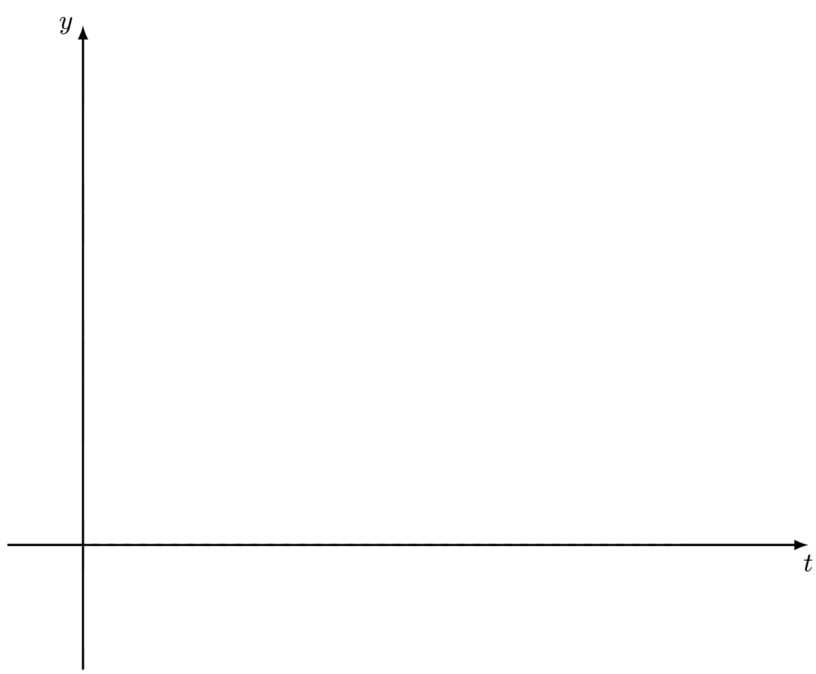
Solution:

(1) Graph ________________________


(2) Find the critical points: ________________________

(3) Find the increasing-decreasing intervals of $f$.


(4) We can skip the concavity regions.
(5) Move the horizontal $y$-axis into a vertical axis, and add a horizontal $t$-axis.

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