1.2. Introduction to Modeling

Section Objective(s):

- Population Models
  - Unlimited Resources
  - Limited Resources
  - Interacting Species

1.2.1. **Population Model with Unlimited Resources.**

Remarks:

- "In the real world food is never unlimited, so unlimited food models are useless."
- However, bacteria in a petri dish have unlimited food—for a while; so the unlimited food model is useful—for a while.
- *Simple models may be good approximations to complex situations for a certain time.*
- *More complicated models are often constructed from simpler ones.*

Case 1: The Unlimited Resources Model. (Review)

Problem: Describe a bacteria population when the have unlimited food, when the population rate change per capita is $r > 0$.

Case 2: The Unlimited Resources with Immigration Model.

Problem: Describe a village population when they have unlimited food, the rate of population growth per capita is 3, and they have an immigration rate of 9 persons per unit time.

Case 3: Radioactive Decay.

Problem: Describe the amount of radioactive material with half-life τ.

Read in Lecture Notes: Using radioactive decay to date remains.

1.2.2. Population Models with Finite Resources.

If the population $P(t)$ is ______________: ______________________________________.

If the population $P(t)$ is ______________: ______________________________________.

Definition 1. The ***logistic equation*** for the function P, which depends on the independent variable t, is

$r > 0$ is the ____________ constant and $P_c > 0$ is the ______________________________.

EXAMPLE 1.2.1: Suppose the function P is solution to the logistic equation

$$P'(t) = r\,P(t)\Big(1 - \frac{P(t)}{P_c}\Big).$$

(a) For what values of P is the population in equilibrium—that is, time independent?
(b) For what values of P is the population increasing in time?
(c) For what values of P is the population decreasing in time?

SOLUTION:

$\triangleleft$

1.2.3. **Interacting Species Model.**

Problem: Write a simple model to describe how rabbits and sheep populations evolve in time when they **compete** on the grass on a particular piece of land.

Solution:

$\lhd$

Definition 2. The ***interacting species equation*** for the functions x and y, which depend on the independent variable t, are

where the constants r_x, r_y and x_c, x_c are positive and α, β are real numbers.

EXAMPLE 1.2.2: The following systems are models of the populations of pair of species that either **compete** for resources (an increase in one species decreases the growth rate in the other) or **cooperate** (an increase in one species increases the growth rate in the other). For each of the following systems identify the independent and dependent variables, the parameters, such as growth rates, carrying capacities, measure of interactions between species. Do the species compete of cooperate?

(a)

$$\frac{dx}{dt} = c_1 x - c_1 \frac{x^2}{K_1} - b_1\, xy$$
$$\frac{dy}{dt} = c_2 y - c_2 \frac{y^2}{K_2} - b_2\, xy.$$

(b)

$$\frac{dx}{dt} = x - \frac{x^2}{5} + 5\, xy$$
$$\frac{dy}{dt} = 2\, y - \frac{y^2}{6} + 2\, xy.$$

SOLUTION:

(a)

(b)

Question: If x are elephants and y are chipmunks, then is $b_1 > b_2$ or $b_2 > b_1$?

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