1.1. Bacteria Reproduce like Rabbits

Section Objective(s):
- Overview of Differential Equations.
- The Discrete Equation.
- The Continuum Equation.
- Summary and Consistency.

1.1.1. **Overview of Differential Equations.**

Remarks:

(a) A differential equation is ____________________, the unknown is ____________________, and both ____________________ and its ____________________ may appear in the equation.

(b) Differential equations are essential for a ____________________ description of nature.

(c) In this section we show where ____________________ equations come from. We focus on a specific problem—a quantitative description of bacteria ____________________ under certain conditions including ____________________ space and food.

1.1.2. The Discrete Equation.

The Problem: We want to know how bacteria grow in time when they have

______________________ space and food supplies. We will find that:

Theorem 1. The population of bacteria $P(n\Delta t)$ after $n \geqslant 1$ time intervals $\Delta t > 0$ is given by the equation

where $r > 0$ is a constant that depends on the particular type of bacteria.

The Experiments:

(1) **First Experiment:** We put an $P(0)$ bacteria in a small region at the center of a petri dish, which is full bacteria food.

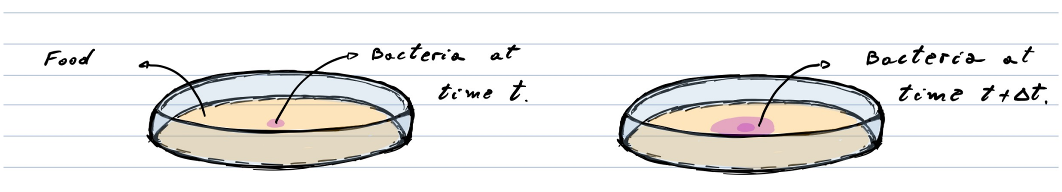

FIGURE 1. Bacteria growth experiment with unlimited food and space.

(2) We measure the bacteria population after regular time intervals.

- The time interval between measurements is ______________________________.
- Denote the bacteria population after n time intervals by ________________________,
- Introduce the initial bacteria population ____________________________,

(3) Our first n measurements are the following,

(4) **Second Experiment:** We reduce the time interval to ________________________ when we take measurements. We find:

where ____________________ .

(5) **Experiment m-th:** We use a time interval ____________________ . We get

where ____________________ . Therefore,

(6) **Summary:**
If we drop the subindex m, we get

where Δt is any time interval, and the discrete equation is then

1.1.3. **Solving the Discrete Equation.**

A discrete equation relates ____________________ with ______________________________.

To solve a discrete equation means to relate ____________________ with _______________.

The discrete equation above can be solved, and the result is summarized below.

Theorem 2. The discrete equation

relating _________________ with ___________________________ has the solution

relating ____________________ with _______________.

(The proof is in the Lecture Notes.)

1.1.4. **The Continuum Equation.**

We study the discrete population equation and its solutions in the

____

Hence ________________. The result is:

Theorem 3. The continuum limit of the discrete equation

is the differential equation

Remark: This differential equation is called ________________________________ equation.

Proof:

1.1.5. **Solving the Continuum Equation.**

Theorem 4. There is only one solution P to the initial value problem

where P_0 is a constant, given by

Proof:

□

1.1.6. **Summary and Consistency.**

We can summarize all this in the following picture

Discrete description	$\Delta t \to 0$	**Continuous description**
$P(n\Delta t) = (1 + r\,\Delta t)\,P((n-1)\Delta t)$	$\longrightarrow$	$P'(t) = r\,P(t)$
$\downarrow$		$\downarrow$
Soving the equation		Solving the equation
$\downarrow$		$\downarrow$
$P(n\Delta t) = (1 + r\,\Delta t)^n\,P_0$	Consistency $\longrightarrow$	$P(t) = P_0\,e^{rt}$

Theorem 5. (Consistency) The ______________________________ of the solutions of the discrete population equations are the solutions of the continuum population equation,

$$P(n\Delta t) = (1 + r\,\Delta t)^n\,P_0 \quad \longrightarrow \quad P(t) = P_0\,e^{rt}.$$

(The proof is in the Lecture Notes.)